



Analytical Simulation of Linear Comparison of Temperature Changes Versus Time in the Heat Equilibrium Process of Mixing Two Liquids

Stephanus Ivan Goenawan

Program Studi Teknik Industri, Fakultas Teknik, Universitas Katolik Indonesia Atma Jaya

E-mail: steph.goenawan@atmajava.ac.id

ABSTRAK

Penjelasan konsep entropy yang menerangkan bahwa peningkatan ketidakteraturan pada suatu sistem tertutup yang bekerja dalam waktu maju adalah berasal dari Hukum Termodinamika kedua. Dalam perkembangannya Hukum Termodinamika 2 diperkaya dengan hadirnya Hukum Termodinamika 2.1 yang menyatakan bahwa proses rerata entropy arah maju akan sama atau lebih kecil daripada arah mundurnya. Pengertian dari proses rerata entropy arah mundur adalah proses rerata entropy arah maju namun bekerja pada fungsi kalor cerminan sebelumnya yaitu fungsi kalor terhadap waktu arah maju. Selanjutnya dengan memanfaatkan Hukum Termodinamika 2.1 ini dapat diketahui hasil simulasi analitik perbandingan tingkat kelinieran perubahan suhu terhadap waktu pada proses aliran panas menuju kesetimbangan termal dalam pencampuran dua zat cair. Hasil perbandingan secara analitik ini menunjukkan salah satu manfaat dari adanya pengembangan Hukum Termodinamika kedua yaitu Hukum Termodinamika 2.1.

Kata kunci :

Entropy; Termodinamika; Suhu; Kesetimbangan Panas.

ABSTRACT

The explanation of the concept of entropy which explains that the increase in disorder in a closed system that works in advancing time comes from the second Law of Thermodynamics. In its development, Law of Thermodynamics 2 was enriched by the presence of Law of Thermodynamics 2.1 which states that the average entropy process in the forward direction will be the same or smaller than in the backward direction. The meaning of the entropy averaging process in the backward direction is that the entropy averaging process is in the forward direction but works on a heat function that mirrors the previous one, namely the heat function over time in the forward direction. Furthermore, by utilizing the Law of Thermodynamics 2.1, the results of analytical simulations can be seen comparing the level of linearity of changes in temperature over time in the process of heat flow towards thermal equilibrium in the mixing of two liquids. The results of this analytical comparison show one of the benefits of developing the second Law of Thermodynamics, namely Law of Thermodynamics 2.1.

Keywords :

Entropy; Thermodynamics; Temperature; Heat Equilibrium.

1. INTRODUCTION

In this complex world, where energy and matter interact, the laws of thermodynamics are the main foundation that guides our understanding of the universe. These laws provide a framework that outlines the basic principles regarding energy, heat, and mechanical work. In this study, the main focus will be on the Second Law of Thermodynamics, which is also known as the Law of Entropy [4]. This law,

elegantly and inevitably, introduces the concept of entropy as a marker of inevitable change in systems [5,6].

The Second Law of Thermodynamics, known as the Law of Entropy, is one of the main foundations in the field of thermodynamics which provides deep insight into the direction of change in a system [7]. The Second Law of Thermodynamics was discovered and formulated as a principle by Rudolf Clausius

in the 19th century. The Law of Entropy provides a critical understanding regarding the impossibility of a system to move spontaneously towards a state with lower entropy [14]. In this context, entropy can be considered as a measure of the disorder of a system, and a deep understanding of this concept opens the door to the development of technology, science, and practical applications in various fields [9].

The Law of Entropy introduces the basic concept that in an isolated system, entropy tends to increase as time goes by [10]. That is, the system tends to switch to a more random and disordered state [11]. This provides a foundation for us to understand the natural processes we witness every day, from physical phenomena to chemical changes that occur in our environment [13]. This concept also plays a crucial role in the exploration of energy and system efficiency, helping us understand the fundamental constraints in energy conversion processes [12]. Understanding the Law of Entropy opens the door to innovation in various fields. In technological development, understanding second thermodynamics has helped design more efficient thermal engines and accelerated progress in the field of renewable energy [15]. On the scientific side, the concept of entropy provides a basis for explaining changes in the state of gases and heat transfer. Successful application of the Law of Entropy not only encouraged the development of modern technology, but also probed the limits of our understanding of the universe.

Understanding the Law of Entropy becomes more important along with the development of technology and scientific applications. This law gives us deep insight into the direction of the universe and the phenomena we observe in everyday life. In this research, we will explore the essence of the Law of Entropy, especially the Law of Thermodynamics 2.1 or Goen's Law of Entropy [3] and its uses. Through a deep understanding of Goen's Law of Entropy, we can explore how we can analyze the comparison of the level of linearity of the temperature change function model over

time in the process towards equilibrium heat flow in mixing two liquids with different temperatures.

2. METHODS

In this theoretical research, the benefits of developing Law of Thermodynamics 2, namely Law of Thermodynamics 2.1 or Goen's Law of Entropy, will be explored in depth. Goen's Entropy Law states that the average value of total entropy in the forward direction is smaller than in the backward direction. So the aim of this research utilizing Goen's Law of Entropy is to use the linearity level comparison technique in the heat flow process model in mixing two liquids that work within a certain time span until they reach thermal equilibrium. By exploring the benefits of Goen's Law of Entropy, it is hoped that we will be able to contribute to the development of science and technology, as well as celebrating the wealth of knowledge produced by Goen's Law of Entropy in our understanding of the universe.

The method used in this research is to use Law of Thermodynamics 2.1, namely comparing the average total entropy in the forward direction with the average total entropy in the backward direction [2]. The total entropy in the backward direction is the total entropy in the forward direction but works on a heat function mirroring the previous one, namely the heat function over time in the forward direction. According to Law of Thermodynamics 2.1, the average total entropy value in the forward direction is equal to or smaller than the average total entropy in the backward direction [3]. In this study, the comparison of these two total entropy averages will be compared through an analytical process [1,8,11]. Mathematically, Law of Thermodynamics 2.1 which states that the average value of total entropy in the working process of a system in the forward direction is smaller or equal to the reverse direction is

$$\frac{\int_0^{t_s} S^+(t).dt}{t_s} \leq \frac{\int_0^{t_s} S^-(t).dt}{t_s}. \quad (1)$$

Furthermore, this research will utilize the Law of Thermodynamics 2.1 to find a model for heat release or heat gain in the form of a function of temperature that is dependent on time in achieving thermal equilibrium. Previously it was possible to prove a formula for the equilibrium temperature of two combined liquids, namely:

$$T_s = \frac{m_a c_a T_a + m_b c_b T_b}{m_a c_a + m_b c_b} \quad (2a)$$

where is the sequence of temperature values: $T_a < T_s < T_b$.

From the second Law of Thermodynamics, the Law of Entropy equation is that the small change in entropy in a system is equal to the small change in heat divided by the current temperature.

$$dS = \frac{dQ(t)}{T(t)} \quad (3)$$

Where in liquids small changes in heat are formulated by

$$dQ(t) = mcdT(t) \quad (4)$$

then from equation (3), the entropy equation (S) becomes

$$S = \int \frac{mc}{T(t)} \frac{dT(t)}{dt} dt \quad (5)$$

3. RESULTS

In this research, a model of heat release or heat gain will be created in the form of a temperature function that is dependent on time in reaching equilibrium which is linear in the time period t_w , which is shown in Figure 1 below:

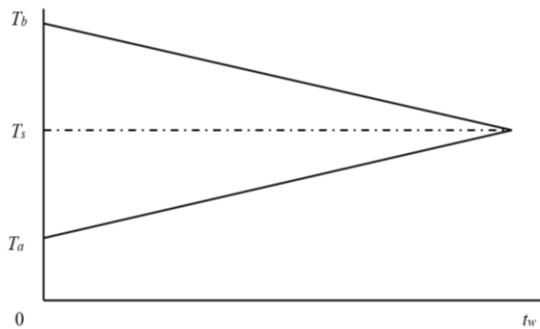


Figure 1. Linear temperature function in the forward direction in achieving equilibrium.

The linear function from T_a to T_s is

$$T_{a \rightarrow s}(t) = g_a t + T_a \quad (6)$$

where the value of $g_a > 0$

$$g_a = \frac{T_s - T_a}{t_w} = \frac{m_b(T_b - T_a)}{(m_a + m_b)t_w} \quad (7)$$

The linear function from T_b to T_s is

$$T_{b \rightarrow s}(t) = g_b t + T_b \quad (8)$$

where the value of $g_b < 0$

$$g_b = \frac{T_s - T_b}{t_w} = -\frac{m_a(T_b - T_a)}{(m_a + m_b)t_w} \quad (9)$$

The Entropy In (S_i) equation from temperature T_a to T_s which is dependent on time when receiving heat is:

$$S_{a \rightarrow s}(t) = \int_0^{t_w} \frac{m_a c_a g_a}{T_{a \rightarrow s}(t)} dt \quad (10)$$

The Entropy Out (S_o) equation from temperature T_b to T_s which is dependent on time when releasing heat is:

$$S_{b \rightarrow s}(t) = \int_0^{t_w} \frac{m_b c_b g_b}{T_{b \rightarrow s}(t)} dt \quad (11)$$

The results of the process of proving the average entropy equation for Entropy In (S_i) in the forward direction in the t_w period $\overline{S_{a \rightarrow s}}$ are :

$$\overline{S_{a \rightarrow s}} = \frac{1}{t_w} \int_0^{t_w} S_{a \rightarrow s}(t) dt = \frac{1}{t_w} \int_0^{t_w} \int_0^t \frac{m_a c_a g_a}{T_{a \rightarrow s}(t)} dt^2 = \frac{1}{t_w} \int_0^{t_w} \int_0^t \frac{m_a c_a g_a d(g_a t + T_a)}{g_a} dt \quad (12a)$$

$$\overline{S_{a \rightarrow s}} = \frac{m_a c_a}{t_w} \int_0^{t_w} [\ln(g_a t + T_a)]_0^t dt = \frac{m_a c_a}{t_w} \int_0^{t_w} [\ln(g_a t + T_a) - \ln(T_a)] dt \quad (12b)$$

$$\overline{S_{a \rightarrow s}} = \frac{m_a c_a}{t_w} \left\{ \int_0^{t_w} \ln(g_a t + T_a) \frac{d(g_a t + T_a)}{g_a} - \int_0^{t_w} \ln(T_a) dt \right\} \quad (12c)$$

$$\overline{S_{a \rightarrow s}} = \frac{m_a c_a}{t_w g_a} \{ [(g_a t + T_a) \ln(g_a t + T_a) - (g_a t + T_a)]_0^{t_w} - g_a \ln(T_a) [t]_0^{t_w} \} \quad (12d)$$

$$\overline{S_{a \rightarrow s}} = \frac{m_a c_a}{t_w g_a} \{ [(T_s \ln T_s - T_s) - (T_a \ln T_a - T_a)] - g_a \ln(T_a) t_w \} \quad (12e)$$

Substitution of eq.(7) into the above equation eq.(12e) can be produced

$$\overline{S_{a \rightarrow s}} = \frac{m_a c_a}{T_s - T_a} \left\{ \ln \left(\frac{T_s T_s}{T_a T_a} \right) - (T_s - T_a) \right\} - m_a c_a \ln(T_a) \quad (12f)$$

$$\overline{S_{a \rightarrow s}} = \frac{m_a c_a}{T_s - T_a} \ln \left(\frac{T_s T_s}{T_a T_a} \right) - m_a c_a (1 + \ln(T_a)) \quad (12g)$$

So the equation $\overline{S_{a \rightarrow s}}$ is:

$$\overline{S_{a \rightarrow s}} = \frac{1}{t_w} \int_0^{t_w} S_{a \rightarrow s}(t) dt = \frac{m_a c_a}{T_s - T_a} \ln \left(\frac{T_s T_s}{T_a T_a} \right) - m_a c_a (1 + \ln T_a) \quad (12h)$$

In the same way, the result of the average entropy equation for Entropy Out (S_o) in the forward direction in the t_w period is

$$\overline{S_{b \rightarrow s}} = \frac{1}{t_w} \int_0^{t_w} S_{b \rightarrow s}(t) dt = \frac{m_b c_b}{T_s - T_b} \ln \left(\frac{T_s T_s}{T_b T_b} \right) - m_b c_b (1 + \ln T_b) \quad (13)$$

The total result of adding the average entropy equations In (S_i) and Out (S_o) in the forward direction in the t_w period is:

$$\begin{aligned} \overline{S_{T+}} &= \overline{S_{a \rightarrow s}} + \overline{S_{b \rightarrow s}} \\ &= \frac{m_a c_a}{T_s - T_a} \ln \left(\frac{T_s T_s}{T_a T_a} \right) + \frac{m_b c_b}{T_s - T_b} \ln \left(\frac{T_s T_s}{T_b T_b} \right) \\ &\quad - m_a c_a (1 + \ln T_a) - m_b c_b (1 + \ln T_b) \end{aligned} \quad (14a)$$

$$(14b)$$

Next, we will look for the equation for the total entropy rate in the backward process, where in this research we will create a model of heat release or heat gain in the form of a function of temperature that is dependent on time after reaching equilibrium, returning to the original temperature, namely the temperature T_a and T_b , which flow linearly over a period of time t_w , which is visible in figure 2 below:

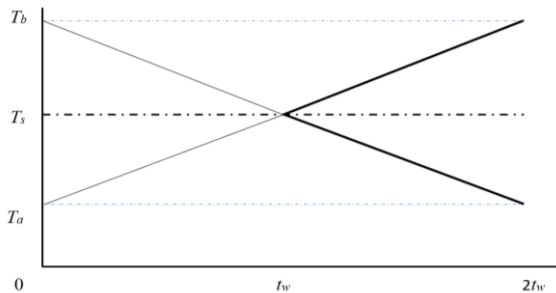


Figure 2. Reverse linear temperature function after reaching equilibrium

From Eq.(6), the linear function from T_s to T_a from t_w to $2t_w$ is

$$T_{s \rightarrow a}(t) = g_a (2t_w - t) + T_a \quad (15)$$

where the value of $g_a > 0$

$$g_a = \frac{T_s - T_a}{t_w} = \frac{m_b (T_b - T_a)}{(m_a + m_b) t_w} \quad (16)$$

Meanwhile, from Eq.(8), the linear function from T_s to T_b is

$$T_{s \rightarrow b}(t) = g_b (2t_w - t) + T_b \quad (17)$$

where the value of $g_b < 0$

$$g_b = \frac{T_s - T_b}{t_w} = -\frac{m_a (T_b - T_a)}{(m_a + m_b) t_w} \quad (18)$$

The Entropy Equation which is related to the reverse process when releasing heat [2]

$$S_{s \rightarrow a}(t) = \int_{t_w}^{2t_w} \frac{m_a c_a g_a}{T_{s \rightarrow a}(t)} dt \quad (19)$$

Entropy equation which is dependent on the reverse process when receiving heat [2]

$$S_{s \rightarrow b}(t) = \int_{t_w}^{2t_w} \frac{m_b c_b g_b}{T_{s \rightarrow b}(t)} dt \quad (20)$$

The results of the process of proving the average entropy equation for the process $s \rightarrow a$ when releasing heat in period t_w $\overline{S_{s \rightarrow a}}$ are:

$$\begin{aligned} \overline{S_{s \rightarrow a}} &= \frac{1}{t_w} \int_{t_w}^{2t_w} S_{s \rightarrow a}(t) dt = \frac{1}{t_w} \int_{t_w}^{2t_w} \int_0^t \frac{m_a c_a g_a}{T_{s \rightarrow a}(t)} dt^2 \\ &= \frac{1}{t_w} \int_{t_w}^{2t_w} \int_0^t \frac{m_a c_a g_a}{g_a (2t_w - t) + T_a} \frac{d(g_a (2t_w - t) + T_a)}{g_a} dt \end{aligned} \quad (21a)$$

$$\begin{aligned} \overline{S_{s \rightarrow a}} &= \frac{m_a c_a}{t_w} \int_{t_w}^{2t_w} [\ln(g_a (2t_w - t) + T_a)]_0^t dt \\ &= \frac{m_a c_a}{t_w} \int_{t_w}^{2t_w} [\ln(g_a (2t_w - t) + T_a) - \ln(g_a (2t_w) + T_a)] dt \end{aligned} \quad (21b)$$

$$\overline{s_{s \rightarrow a}} = \frac{m_a c_a}{t_w} \left\{ \int_{t_w}^{2t_w} \ln(g_a(2t_w - t) + T_a) \frac{d(g_a(2t_w - t) + T_a)}{g_a} - \int_{t_w}^{2t_w} \ln(2g_a t_w + T_a) dt \right\} \quad (21c)$$

$$\overline{s_{s \rightarrow a}} = \frac{m_a c_a}{t_w g_a} \left\{ [(g_a(2t_w - t) + T_a) \ln(g_a(2t_w - t) + T_a) - (g_a(2t_w - t) + T_a)]_{t_w}^{2t_w} - g_a \ln(2g_a t_w + T_a) [t]_{t_w}^{2t_w} \right\} \quad (21d)$$

$$\overline{s_{s \rightarrow a}} = \frac{m_a c_a}{t_w g_a} \left\{ [(T_a \ln T_a - T_a) - (T_s \ln T_s - T_s)] - g_a \ln(2g_a t_w + T_a) t_w \right\} \quad (21e)$$

Substituting Eq.(7) into the above equation eq.(21e) results

$$\overline{s_{s \rightarrow a}} = \frac{m_a c_a}{T_s - T_a} \left\{ \ln \left(\frac{T_a T_a}{T_s T_s} \right) + (T_s - T_a) \right\} - m_a c_a \ln(2g_a t_w + T_a) \quad (21f)$$

$$\overline{s_{s \rightarrow a}} = - \frac{m_a c_a}{T_s - T_a} \ln \left(\frac{T_s T_s}{T_a T_a} \right) + m_a c_a (1 - \ln(2g_a t_w + T_a)) \quad (21g)$$

So the equation $\overline{s_{s \rightarrow a}}$ is:

$$\overline{s_{s \rightarrow a}} = \frac{1}{t_w} \int_{t_w}^{2t_w} s_{s \rightarrow a}(t) dt = - \frac{m_a c_a}{T_s - T_a} \ln \left(\frac{T_s T_s}{T_a T_a} \right) + m_a c_a (1 - \ln(2g_a t_w + T_a)) \quad (21h)$$

Then in the same way, the result of the average entropy equation for the process $s \rightarrow b$ when releasing heat in period t_w is

$$\overline{s_{s \rightarrow a}} = \frac{1}{t_w} \int_{t_w}^{2t_w} s_{s \rightarrow a}(t) dt = - \frac{m_a c_a}{T_s - T_a} \ln \left(\frac{T_s T_s}{T_a T_a} \right) + m_a c_a (1 - \ln(2g_a t_w + T_a)) \quad (22)$$

The total result of adding the average entropy equations for the processes $s \rightarrow a$ and $s \rightarrow b$ in period t_w is:

$$\overline{s_{T-}} = \overline{s_{s \rightarrow a}} + \overline{s_{s \rightarrow b}} \quad (23a)$$

$$\overline{s_{T-}} = - \frac{m_a c_a}{T_s - T_a} \ln \left(\frac{T_s T_s}{T_a T_a} \right) - \frac{m_b c_b}{T_s - T_b} \ln \left(\frac{T_s T_s}{T_b T_b} \right) + m_a c_a (1 - \ln(2g_a t_w + T_a)) + m_b c_b (1 - \ln(2g_b t_w + T_b)) \quad (23b)$$

Next we will compare the two total entropy equations eq.(14) with eq.(23) according to the Law of Thermodynamics 2.1 [2,3], namely:

$$\overline{s_{T+}} < \overline{s_{T-}} \quad (24)$$

By substituting eq.(14) and eq.(23) into eq.(24) the following inequality results will be obtained:

$$\frac{m_a c_a}{T_s - T_a} \ln \left(\frac{T_s T_s}{T_a T_a} \right) + \frac{m_b c_b}{T_s - T_b} \ln \left(\frac{T_s T_s}{T_b T_b} \right) - m_a c_a (1 + \ln T_a) - m_b c_b (1 + \ln T_b) < - \frac{m_a c_a}{T_s - T_a} \ln \left(\frac{T_s T_s}{T_a T_a} \right) - \frac{m_b c_b}{T_s - T_b} \ln \left(\frac{T_s T_s}{T_b T_b} \right) + m_a c_a (1 - \ln(2g_a t_w + T_a)) + m_b c_b (1 - \ln(2g_b t_w + T_b)) \quad (25)$$

Some terms on the right side are moved to the left side to become:

$$\frac{2m_a c_a}{T_s - T_a} \ln \left(\frac{T_s T_s}{T_a T_a} \right) + \frac{2m_b c_b}{T_s - T_b} \ln \left(\frac{T_s T_s}{T_b T_b} \right) - 2m_a c_a - m_a c_a \ln T_a - 2m_b c_b - m_b c_b \ln T_b < -m_a c_a \ln(2g_a t_w + T_a) - m_b c_b \ln(2g_b t_w + T_b) \quad (26)$$

Some terms on the left side are moved to the right side to become:

$$\frac{2m_a c_a}{T_s - T_a} \ln \left(\frac{T_s T_s}{T_a T_a} \right) + \frac{2m_b c_b}{T_s - T_b} \ln \left(\frac{T_s T_s}{T_b T_b} \right) - 2m_a c_a - 2m_b c_b < -m_a c_a \ln(2g_a t_w + T_a) + m_a c_a \ln T_a - m_b c_b \ln(2g_b t_w + T_b) + m_b c_b \ln T_b \quad (27)$$

Then all the position fields are changed to:

$$m_a c_a \ln(2g_a t_w + T_a) - m_a c_a \ln T_a + m_b c_b \ln(2g_b t_w + T_b) - m_b c_b \ln T_b < 2m_a c_a + 2m_b c_b - \frac{2m_a c_a}{T_s - T_a} \ln \left(\frac{T_s T_s}{T_a T_a} \right) - \frac{2m_b c_b}{T_s - T_b} \ln \left(\frac{T_s T_s}{T_b T_b} \right) \quad (28)$$

Applying the distributive property to the term on the left becomes:

$$m_a c_a (\ln(2g_a t_w + T_a) - \ln T_a) + m_b c_b (\ln(2g_b t_w + T_b) - \ln T_b) < 2m_a c_a + 2m_b c_b - \frac{2m_a c_a}{T_s - T_a} \ln \left(\frac{T_s T_s}{T_a T_a} \right) - \frac{2m_b c_b}{T_s - T_b} \ln \left(\frac{T_s T_s}{T_b T_b} \right) \quad (29)$$

Then proceed with the logarithmic property to become:

$$m_a c_a \ln \left(\frac{(2g_a t_w + T_a)}{T_a} \right) + m_b c_b \ln \left(\frac{(2g_b t_w + T_b)}{T_b} \right) < 2m_a c_a + 2m_b c_b - \frac{2m_a c_a}{T_s - T_a} \ln \left(\frac{T_s T_s}{T_a T_a} \right) - \frac{2m_b c_b}{T_s - T_b} \ln \left(\frac{T_s T_s}{T_b T_b} \right) \quad (30)$$

If applied in the special case of $m_a c_a = m_b c_b$, then from eq.(2) we can obtain the equation relationship $T_s - T_a = T_b - T_s$, so that eq.(30) can be simplified to:

$$\ln \left(\frac{(2g_a t_w + T_a)}{T_a} \right) + \ln \left(\frac{(2g_b t_w + T_b)}{T_b} \right) < 4 - \frac{2}{T_s - T_a} \ln \left(\frac{T_s T_s}{T_a T_a} \right) - \frac{2}{T_s - T_b} \ln \left(\frac{T_s T_s}{T_b T_b} \right) \quad (31)$$

By logarithmic properties the terms on the left can be combined to form:

$$\ln \left(\frac{(2g_a t_w + T_a)(2g_b t_w + T_b)}{T_a T_b} \right) < 4 - \frac{2}{T_s - T_a} \ln \left(\frac{T_s T_s}{T_a T_a} \right) - \frac{2}{T_s - T_b} \ln \left(\frac{T_s T_s}{T_b T_b} \right) \quad (32)$$

Then raise all sides to the power of the natural number e to become:

$$\frac{(2g_a t_w + T_a)(2g_b t_w + T_b)}{T_a T_b} < \exp\left(4 - \frac{2}{T_s - T_a} \ln\left(\frac{T_s^{T_s}}{T_a^{T_a}}\right) - \frac{2}{T_s - T_b} \ln\left(\frac{T_s^{T_s}}{T_b^{T_b}}\right)\right) \quad (33)$$

By substituting eq.(7) and eq.(9) into eq.(33) above we get the results:

$$\frac{(2T_s - T_a)(2T_s - T_b)}{T_a T_b} < \exp\left(4 - \frac{2}{T_s - T_a} \ln\left(\frac{T_s^{T_s}}{T_a^{T_a}}\right) + \frac{2}{T_b - T_s} \ln\left(\frac{T_s^{T_s}}{T_b^{T_b}}\right)\right) \quad (34)$$

Then by using logarithmic properties, the right side of the equation can be simplified again to:

$$\frac{(2T_s - T_a)(2T_s - T_b)}{T_a T_b} < \exp\left(4 + \frac{2}{T_s - T_a} T_a \ln T_a - \frac{2}{T_b - T_s} T_b \ln T_b\right) \quad (35)$$

Furthermore, with the distributive property and the equation value $T_s - T_a = T_b - T_s$, the terms on the right can be combined to become:

$$\frac{(2T_s - T_a)(2T_s - T_b)}{T_a T_b} < \exp\left(4 + \frac{2}{T_s - T_a} (T_a \ln T_a - T_b \ln T_b)\right) \quad (36)$$

Using the results of the inequality in eq.(36), for example, in discussing the value of mixing with water, the heat capacity value is $c_a = c_b$ and both thermos contain water with the same mass $m_a = m_b$, and the temperature of the water in the thermos is $T_a = 20C$ and $T_b = 80C$, as in seen in the figure 3 below:

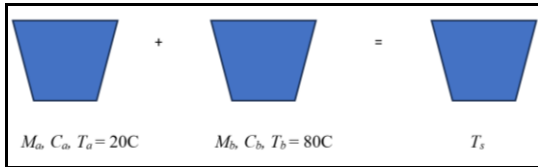


Figure 3. Mixing two liquids with the same capacity

The mixture temperature value T_s is:

$$T_s = \frac{m_a T_a + m_b T_b}{m_a + m_b} = \frac{T_a + T_b}{2} = 50C \quad (37)$$

Then from the initial data $T_a = 20C$ and $T_b = 80C$ as well as the results of the equilibrium temperature of the mixture eq.(37) $T_s = 50C$ are substituted into the eq.(36) the inequality results below are obtained.

$$\frac{(100 - 20)(100 - 80)}{20 \cdot 80} < \exp\left(4 + \frac{2}{50 - 20} (20 \ln 20 - 80 \ln 80)\right) \quad (38)$$

After carrying out the calculation, the resulting inequality value above eq.(38) becomes: $1 < 2,09 \cdot 10^{-7}$, it turns out that the result of this calculation does not meet the requirements of eq.(36) because the value should be $1 > 2,09 \cdot 10^{-7}$. So analytically it has been proven that a linear model for the function of time to reach equilibrium is theoretically impossible and of course in practice in this case it will definitely never happen.

As a comparison in the second case, using the inequality results in eq.(36), for example in discussing the value of mixing with water, the heat capacity value $c_a = c_b$ and both thermos contain water with the same mass $m_a = n \cdot m_b$, as well as the temperature of the water in the thermos $T_a = T_b/n$, for example $n = 4$ then we can get $T_a = 20C$ and $T_b = 80C$, as shown in Figure 4 below

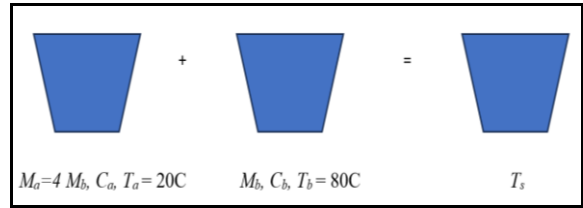


Figure 4. Mixing two liquids with $m_a = n \cdot m_b$ and $T_a = T_b/n$.

The mixture temperature value T_s is:

$$T_s = \frac{m_a T_a + m_b T_b}{m_a + m_b} = \frac{nm_b(T_b/n) + m_b T_b}{nm_b + m_b} = \frac{2T_b}{(n+1)} = \frac{2 \cdot 80}{(4+1)} = 32C \quad (39)$$

Then from the initial data $T_a = 20C$ and $T_b = 80C$ as well as the results of the equilibrium temperature of the mixture eq.(39) $T_s = 32C$ is substituted into eq.(36) the inequality results below will be obtained

$$\frac{(64 - 20)(64 - 80)}{20 \cdot 80} < \exp\left(4 + \frac{2}{32 - 20} (20 \ln 20 - 80 \ln 80)\right) \quad (40)$$

After carrying out the calculation, the resulting inequality value above eq.(40) becomes: $-0,44 < 5,0 \cdot 10^{-20}$, it turns out

that the results of this calculation meet the requirements of eq.(36) because the value of -0.44 is more smaller than $5,0 \cdot 10^{-20}$. So analytically it has been proven that a linear model for the function of time in reaching equilibrium is theoretically possible and of course in practice it can be approached that the requirements for this second case are that the heat flow process can be proven to be closer to linear than the first case. The advantage of this research is the ability to analytically model the function of time versus temperature as a heat flow until it reaches thermal equilibrium with respect to the initial state of the two liquids, whereas in practice it is not possible to model this function by direct measurement because the two liquids are directly mixed together. In practice, what can be done to measure the function of the heat flow against time until it reaches thermal equilibrium is to measure the temperature of one of the liquids before it is mixed, then after the combination of the second liquid, the temperature change against time is measured until it reaches thermal equilibrium.

CONCLUSION

Theoretically, using the Law of Thermodynamics 2.1 or Goen's Law of Entropy, analytical measurements can be made of the comparative value of the level of linearity of the heat flow process when it reaches thermal equilibrium when mixing liquids. The novelty of this research is that it proves that:

- a. Law of Thermodynamics 2.1 or Goen's Law of Entropy can be useful for the progress of Thermodynamics Science.
- b. Utilization for the first time of the Law of Thermodynamics 2.1 in determining the ratio of the degree of linearity of a heat flow to reach the thermal equilibrium temperature in a system mixing two liquids.

The novelty of this research is analytical proof for the first time using the Law of Thermodynamics 2.1 on the comparison of the level of linearity of heat flow when reaching thermal equilibrium on mixing two liquids.

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