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Effective Solutions for Moving Objects in Acceleration Functions and Their Reflections Using Goen's Distance Formula

Stephanus Ivan Goenawan[1]
[1]Program Studi Teknik Industri, Fakultas Teknik, Universitas Katolik Indonesia Atma Jaya
E-mail: steph.goenawan@atmajaya.ac.id

ABSTRAK

Formula jarak Goen adalah formula yang efektif untuk menghitung jarak dari percepatan yang mempunyai pola sebagai fungsi simetri terhadap perubahan waktu. Pada formula jarak Goen dimana fungsi percepatan terhadap waktu berpola simetris dapat diselesaikan cukup dengan sekali integral total kemudian dikalikan dengan setengah dari waktu total. Fungsi percepatan simetris dapat dibangun dari gabungan antara fungsi tertentu dengan cerminannya. Dalam penelitian ini metode pembuktian formula jarak Goen dilakukan secara matematis menggunakan integral rerata bertingkat satu dan dua dari fungsi simetris. Keunggulan dari formula jarak Goen dibandingkan dengan formula jarak konvensional adalah dalam mencari jarak tempuh benda yang bergerak dengan fungsi percepatan simetri akan lebih efektif karena cukup sekali melakukan integral dibandingkan harus dua kali integral seperti yang dilakukan cara konvensional.

Kata kunci :
Formula Jarak Goen; Fungsi Percepatan Simetris; Integral Rerata

ABSTRACT

Goen's distance formula effectively calculates the distance from acceleration, which has a pattern as a symmetric function of time changes. In Goen's distance formula, where the acceleration function against time has a symmetric pattern, it can be solved simply by performing a total integral and multiplying it by half of the total time. Symmetric acceleration functions can be constructed by combining certain functions with their reflections. In this study, the method of proving Goen's distance formula is carried out mathematically using single and double-level average integrals of symmetric functions. The advantage of Goen's distance formula compared to the conventional distance formula is that in finding the distance traveled by an object moving with a symmetric acceleration function, it will be more effective because it is sufficient to perform an integral once compared to having to do two integrals as done in the conventional method.

Keywords **:**
Goen Distance Formula; Mean Integral; Symmetric Acceleration Function.

1. INTRODUCTION

In the field of kinematic physics with recent research references on kinematics and distance calculations, it can be seen that the double integral of the acceleration function is the distance traveled [5-14]. However, if the acceleration function of the object's motion has a symmetrical pattern with respect to changes in time [6, 7], then solving the distance traveled by the object's motion can be more effective by using the Goen distance formula [4]. Therefore, implications of the Goen distance formula is an easier formula than the conventional formula if the acceleration is a symmetric function. This Goen distance formula is a distance formula where the symmetric acceleration function can be solved with just one integral and then multiplied by half the total time.

The method of proving this Goen distance formula is done mathematically using a single-level average integral which has the same value as the second average integral when working on a symmetric function [3]. The advantage of the Goen distance formula compared to conventional methods in finding the distance which is a double integral of acceleration is that in finding this distance traveled, the symmetric acceleration function is only integrated once so that it will be more effective and easier in calculating a distance from the motion of an object. Moreover, if the acceleration pattern is a non-linear function and its reflection will be much easier if solved using this Goen distance formula.

2. METHODS

Proof of Goen's distance formula, can first be done through a discrete equation of the high-order average, in the condition that the first-order average value is the same as the second-order average, which is then done continuously [3]. Previously, it should be noted that the average value of a data is the total sum of the data values divided by the total number of data. So the definition of a high-order average of a data is the total sum of the data divided by the total number of data levels [1, 2]. In this study, the method used in proving the Goen's distance formula is to create a continuous equation that the first and second-order average values will be the same if done on a symmetric function. Discrete formula for calculating the first-order average

$$\overline{f}^{(1)} = \frac{f_0 + f_1 + \cdots + f_T}{T+1} \tag{1}$$

or abbreviated to:

$$\overline{f}^{(1)} = \frac{\sum_{i=0}^{T} f_i}{T+1} \tag{2}$$

and the continuity equation is:

$$\overline{f}^{(1)} = \frac{\int_0^T f_s(t) \cdot dt}{T} \tag{3}$$

Discrete formula for calculating the second order average:

$$\overline{f}^{(2)} = \frac{f_0 + (f_0 + f_1) + \cdots + (f_0 + \cdots + f_T)}{\left(\frac{(T+1)\cdot(T+2)}{2}\right)} \tag{4}$$

or abbreviated to:

$$\overline{f}^{(2)} = \frac{\sum_{t=0}^{T}\sum_{i=0}^{t} f_i}{\left(\frac{(T+1)\cdot(T+2)}{2}\right)} \tag{5}$$

and the continuity equation can be rewritten as:

$$\overline{f}^{(2)} = \frac{\int_{t=0}^{T}\int_0^t f_s(t)\cdot dt \cdot dt}{\frac{T^2}{2}} \tag{6}$$

Furthermore, from Goen's double integral [3, 4] it can be seen that the first and second order continuous averages on the symmetric acceleration function $f_s(t)$ are the same, namely $\overline{f}^{(2)} = \overline{f}^{(1)}$ then:

$$\frac{\int_{t=0}^{T}\int_0^t f_s(t)\cdot dt \cdot dt}{\frac{T^2}{2}} = \frac{\int_0^T f_s(t)\cdot dt}{T} \tag{7}$$

So that the results are obtained:

$$\int_{t=0}^{T}\int_0^t f_s(t)\cdot dt \cdot dt = \frac{T}{2}\cdot \int_0^T f_s(t)\cdot dt \tag{8}$$

So if the conventional distance formula L_k is:

$$L_k(T) = \int_{t=0}^{T}\int_0^t f_s(t)\cdot dt\cdot dt \tag{9}$$

then the Goen distance formula L_g is:

$$L_g(T) = \frac{T}{2}\cdot\int_0^T f_s(t)\cdot dt \tag{10}$$

If the symmetric acceleration function is constructed from an arbitrary function $f^+(t)$ over a time period from 0 to T, and its mirror function $f^-(t) = f^+(T-t)$ over a time period from T to $2.T$ then the Goen distance formula eq.(10) can be rewritten as:

$$L_g(2T) = T\cdot\int_0^{2.T}\left(f^+(t)|_0^T + f^+(2\cdot T - t)|_T^{2.T}\right)\cdot dt \tag{11}$$

which can be distributed into the Goen distance formula again, namely:

$$L_g(2T) = T\cdot\left(\int_0^T f^+(t)\cdot dt + \int_T^{2.T} f^+(2\cdot T - t)\cdot dt\right) \tag{12}$$

Meanwhile, if the calculation uses the conventional distance formula, eq.(9) can be rewritten as:

$$L_k(2T) = \int_0^{2.T}\int_0^t \left(f^+(t)|_0^T + f^+(2\cdot T - t)|_T^{2.T}\right)\cdot dt\cdot dt \tag{13}$$

which can be redistributed into the equation:

$$L_k(2T) = \int_0^{2.T}\int_0^{t}(f^+(t)|_0^T)\cdot dt\cdot dt + \int_T^{2.T}\int_T^{t}(f^+(2\cdot T - t)|_T^{2.T})\cdot dt\cdot dt \quad (14)$$

Where the first term in eq.(14) is also decomposed again, so that the conventional distance formula becomes:

$$L_k(2T) = \int_0^{T}\int_0^{t} f^+(t)\cdot dt\cdot dt + \int_T^{2.T}\int_0^{T} f^+(t)\cdot dt\cdot dt + \int_T^{2.T}\int_T^{t} f^+(2\cdot T - t)\cdot dt\cdot dt \quad (15)$$

After the conventional distance formula eq.(15) and the Goen distance formula eq.(12) are obtained, the following will discuss the results of the application of the Goen distance formula which is more effective compared to the conventional distance formula.

3. RESULTS

The application of Goen's distance formula, eq.(10), to an acceleration function together with its reflection in determining the total distance, the calculation process will be more effective and efficient than the conventional method.

The first case of constant acceleration $f_s(t) = k$, figure 1, where the calculation of the total distance $L = L_g$ will be solved using the Goen and conventional distance formulas.

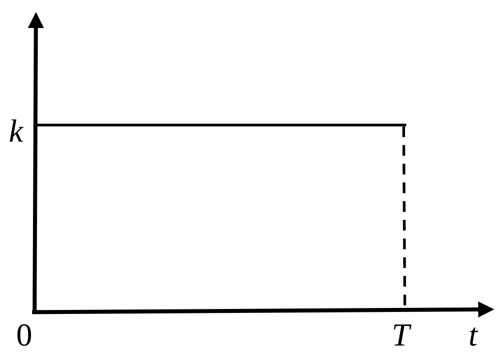

Figure 1. graph of constant acceleration $f_s(t) = k$

The results of the calculation of the total distance from time 0 to T using the Goen formula are:

$$L_g = \frac{T}{2}\cdot\int_0^T f_s(t).\,dt = \frac{T}{2}\cdot\int_0^T k.\,dt = \frac{T}{2}\cdot[k.t]_0^T = k\cdot\frac{T^2}{2} \quad (16)$$

The result of calculating the total distance using the conventional formula:

$$L_k = \int_0^T\int_0^t f_s(t).\,dt^2 = \int_0^T\int_0^t k.\,dt^2 = \int_0^T k.t.\,dt = \frac{k}{2}\cdot[t^2]_0^T = k\cdot\frac{T^2}{2} \quad (17)$$

So it is proven that the final result of the two integral methods above is the same.

The second case of linear acceleration $f^+(t) = t$ with its mirror function $f^-(t) = f^+(T-t) = T - t$, and the distance symbol as $L = L_g$. The symmetry function of linear acceleration with its mirror is $f_s(t) = t|_0^T + (2.T - t)|_T^{2.T}$, figure 2.

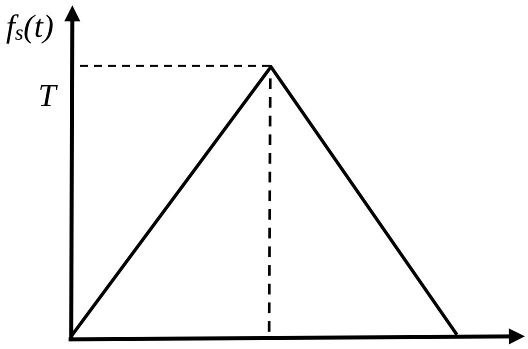

Figure 2. graph of linear acceleration with its mirror function $f_s(t) = t|_0^T + (2.T - t)|_T^{2.T}$

The result of calculating the total distance from time 0 to 2.T using Goen's formula is:

$$L_g = \int_0^{2\cdot T}\int_0^t f_s(t).\,dt^2 = \frac{2\cdot T}{2}\cdot\int_0^{2\cdot T}(t|_0^T + (2.T - t)|_T^{2.T}).\,dt \quad (18)$$

$$L_g = T\cdot\left(\int_0^T t\,dt + \int_T^{2\cdot T}(2.T - t)\,dt\right) = T\cdot\left(\left[\frac{t^2}{2}\right]_0^T - \left[\frac{(2.T-t)^2}{2}\right]_T^{2.T}\right) \quad (19)$$

$$L_g = \frac{T}{2}\cdot\left((T^2 - 0^2) - (0^2 - T^2)\right) = \frac{T}{2}\cdot 2.T^2 = T^3 \quad (20)$$

The result of calculating the total distance from time 0 to $2.T$ using the conventional formula L_k is:

$$L_k = \int_0^{2T}\int_0^t f_s(t).dt^2 = \int_0^{2T}\int_0^t \left(t|_0^T + (2.T-t)|_T^{2.T}\right).dt^2 \quad (21)$$

$$L_k = \int_0^{2T}\int_0^t t|_0^T.dt^2 + \int_T^{2T}\int_T^t (2.T-t).dt^2 = \int_0^T \frac{t^2}{2}.dt + \int_T^{2T}\frac{T^2}{2}.dt + \int_T^{2T}\left(-\frac{(2.T-t)^2}{2}+\frac{T^2}{2}\right).dt \quad (22)$$

$$L_k = \left[\frac{t^3}{6}\right]_0^T + \left[\frac{T^2}{2}.t\right]_T^{2.T} + \left[\frac{(2.T-t)^3}{6}+\frac{T^2}{2}.t\right]_T^{2.T} = \frac{T^3}{6}+\frac{T^3}{2}+\left(T^3-\frac{T^3}{6}-\frac{T^3}{2}\right) = T^3 \quad (23)$$

So it is proven that the final result of the two integral methods above is the same.

The third case of linear acceleration $f^+(t) = p.t$, with its mirror function $f^-(t) = f^+(T-t) = p.(T-t)$, the value of p is constant and the distance symbol is $L = L_g$. The symmetry function of linear acceleration with its mirror is $f_s(t) = p.t|_0^T + p.(2.T-t)|_T^{2T}$, figure 3.

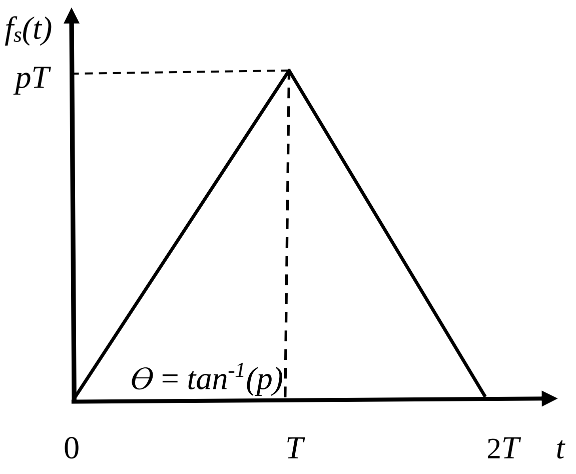

Figure 3. graph of linear acceleration with its mirror function $f_s(t) = p.t|_0^T + p.(2.T-t)|_T^{2T}$.

The result of calculating the total distance from time 0 to $2.T$ using the Goen distance formula is:

$$L_g = \int_0^{2.T}\int_0^t f_s(t).dt^2 = \frac{2\cdot T}{2}\cdot\int_0^{2.T}\left(p.t|_0^T + p.(2.T-t)|_T^{2.T}\right).dt \quad (24)$$

$$L_g = p.T\cdot\left(\int_0^T t\,dt + \int_T^{2.T}(2.T-t)\,dt\right) = T\cdot\left(\left[\frac{t^2}{2}\right]_0^T - \left[\frac{(2.T-t)^2}{2}\right]_T^{2.T}\right) \quad (25)$$

$$L_g = \frac{p.T}{2}\cdot\left((T^2-0^2)-(0^2-T^2)\right) = \frac{p.T}{2}\cdot 2.T^2 = p.T^3 \quad (26)$$

Meanwhile, the results of the total distance calculation from time 0 to $2.T$ using the conventional formula L_k look longer, namely:

$$L_k = \int_0^{2T}\int_0^t f_s(t).dt^2 = \int_0^{2T}\int_0^t \left(p.t|_0^T + p.(2.T-t)|_T^{2.T}\right).dt^2 \quad (27)$$

$$L_k = \int_0^{2T}\int_0^t p.t|_0^T.dt^2 + \int_T^{2T}\int_T^t p.(2.T-t).dt^2 \quad (28)$$

$$L_k = \int_0^T \frac{p.t^2}{2}.dt + \int_T^{2T}\frac{p.T^2}{2}.dt + \int_T^{2T} p.\left(-\frac{(2.T-t)^2}{2}+\frac{T^2}{2}\right).dt \quad (29)$$

$$L_k = p.\left[\frac{t^3}{6}\right]_0^T + p.\left[\frac{T^2}{2}.t\right]_T^{2.T} + p.\left[\frac{(2.T-t)^3}{6}+\frac{T^2}{2}.t\right]_T^{2.T} = \frac{p.T^3}{6}+\frac{p.T^3}{2}+p.\left(T^3-\frac{T^3}{6}-\frac{T^3}{2}\right) = p.T^3 \quad (30)$$

So it is proven that the final result of the two integral methods above is the same.

The fourth case of linear acceleration $f^+(t) = p.t + q$, with its mirror function $f^-(t) = f^+(T-t) = p.(T-t)$, the values of p and q are constant and the distance symbol is $L = L_g$. The symmetry function of linear acceleration with its mirror is $f_s(t) = p.t + q|_0^T + p.(2.T-t) + q|_T^{2.T}$, figure 4.

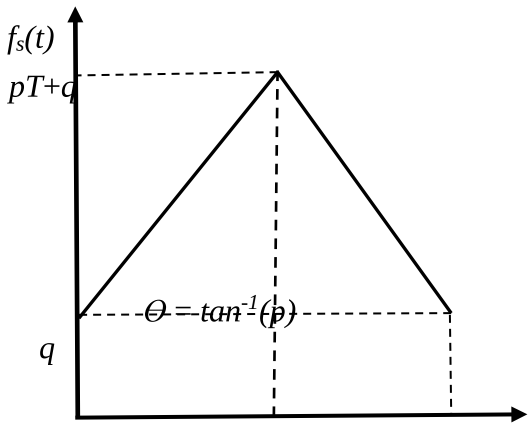

Figure 4. graph of linear acceleration with its mirror function $f_s(t) = p.t + q|_0^T + p.(2.T-t) + q|_T^{2.T}$.

The result of calculating the total distance from time 0 to $2.T$ using the Goen distance formula is:

$$L_g = \int_0^{2T} \int_0^t f_s(t).\, dt^2 = T \cdot \int_0^T f_s(t).\, dt \quad (31)$$

$$L_g = T \cdot \int_0^{2\cdot T} \left(p.t + q|_0^T + p.(2.T - t) + q|_T^{2.T}\right). dt \quad (32)$$

$$L_g = T \cdot \left(\int_0^T (p.t + q) dt + \int_T^{2\cdot T} (p.(2.T - t) + q)\, dt \right) \quad (33)$$

$$L_g = T \cdot \left(\left[p.\frac{t^2}{2} + q.t \right]_0^T + \left[-p.\frac{(2.T-t)^2}{2} + q.t \right]_T^{2.T} \right) \quad (34)$$

$$L_g = T \cdot \left\{ \left(\frac{p}{2} \cdot T^2 + q \cdot T \right) + \left(\frac{p}{2} \cdot T^2 + q \cdot T \right) \right\} \quad (35)$$

$$L_g = T.(p.T^2 + 2.q.T) = T^2.(p.T + 2.q) \quad (36)$$

Meanwhile, the results of the total distance calculation from time 0 to $2.T$ using the conventional formula L_k is:

$$L_k = \int_0^{2T} \int_0^t f_s(t).\, dt^2 = \int_0^{2T} \int_0^t \left(p.t + q|_0^T + p.(2.T - t) + q|_T^{2.T}\right). dt^2 \quad (37)$$

$$L_k = \int_0^{2T} \int_0^t p.t + q|_0^T . dt^2 + \int_T^{2T} \int_T^t (p.(2.T - t) + q). dt^2 \quad (38)$$

$$L_k = \int_0^T \left(\frac{p.t^2}{2} + q \cdot t \right). dt + \int_T^{2T} \left(\frac{p.T^2}{2} + q \cdot T \right). dt + \int_T^{2T} p.\left(-\frac{(2.T - t)^2}{2} + \frac{T^2}{2} \right) + q.(t - T). dt \quad (39)$$

$$L_k = \left[\frac{p.t^3}{6} + \frac{q.t^2}{2} \right]_0^T + \left[\frac{p.T^2}{2} \cdot t + q.T.t \right]_T^{2.T} + \left[p.\left(\frac{(2.T-t)^3}{6} + \frac{T^2}{2} \cdot t \right) + q.\frac{(t - T)^2}{2} \right]_T^{2.T} \quad (40)$$

$$L_k = \left(\frac{p.T^3}{6} + \frac{q.T^2}{2} \right) + \left(\frac{p.T^3}{2} + q.T^2 \right) + p.\left(-\frac{T^3}{6} + \frac{T^3}{2} \right) + q.\frac{T^2}{2} \quad (41)$$

$$L_k = p.T^3 + 2.q.T^2 = T^2.(p.T + 2.q) \quad (42)$$

So it is proven that the final result of the two integral methods above is the same.

The fifth case of nonlinear acceleration $f^+(t) = p.t^r$ with its mirror function $f^-(t) = f^+(T - t) = p.(T - t)^r$, the values of r and p are constant ($r \neq -1$), and the distance symbol is $L = L_g$. The symmetry function of nonlinear acceleration is $f_s(t) = p.t^r|_0^T + p.(2.T - t)^r|_T^{2.T}$, figure 5.

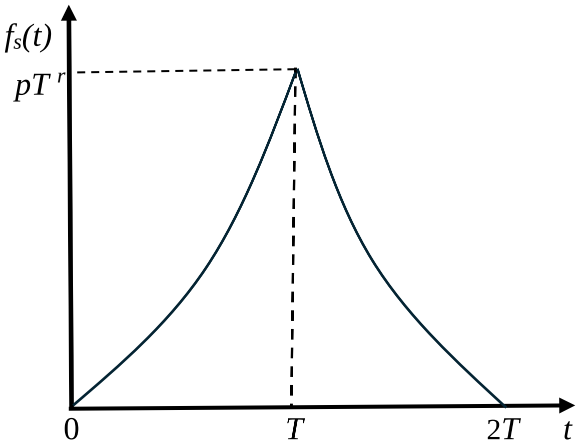

Figure 5. graph of nonlinear acceleration with its mirror function $f_s(t) = p.t^r|_0^T + p.(2.T - t)^r|_T^{2.T}$.

The result of calculating the total distance from time 0 to $2.T$ using the Goen distance formula is:

$$L_g = \int_0^{2\cdot T} \int_0^t f_s(t).\, dt^2 = T \cdot \int_0^{2\cdot T} \left(p.t^r|_0^T + p.(2.T - t)^r|_T^{2.T}\right). dt \quad (43)$$

$$L_g = T \cdot \left(\int_0^T (p.t^r) dt + \int_T^{2\cdot T} (p.(2.T - t)^r)\, dt \right) \quad (44)$$

$$L_g = p.T.\left(\left[\frac{t^{r+1}}{r+1} \right]_0^T - \left[\frac{(2.T-t)^{r+1}}{r+1} \right]_T^{2\cdot T} \right) \quad (45)$$

$$L_g = T.p.\left(\left[\frac{T^{r+1}}{r+1} - 0 \right] - \left[0 - \frac{T^{r+1}}{r+1} \right] \right) = 2.p.T.\left(\frac{T^{r+1}}{r+1} \right) = \frac{2 \cdot p \cdot T^{r+1}}{r + 1} \quad (46)$$

Compare the results of the total distance calculation from time 0 to $2.T$ using the conventional formula L_k is:

$$L_k = \int_0^{2T} \int_0^t f_s(t).\, dt^2 = \int_0^{2T} \int_0^t \left(p.t^r|_0^T + p.(2.T - t)^r|_T^{2.T}\right). dt^2 \quad (47)$$

$$L_k = \int_0^{2T} \int_0^t p.t^r|_0^T . dt^2 + \int_T^{2T} \int_T^t (p.(2.T - t)^r). dt^2 \quad (48)$$

$$L_k = \int_0^T \left(p.\frac{t^{r+1}}{r+1} \right). dt + \int_T^{2T} \left(p.\frac{T^{r+1}}{r+1} \right). dt + \int_T^{2T} p \cdot \left(\frac{-(2.T - t)^{r+1}}{r+1} + \frac{T^{r+1}}{r+1} \right). dt \quad (49)$$

$$L_k = \left[p \cdot \frac{t^{r+2}}{(r+2).(r+1)} \right]_0^T + \left[p \cdot \frac{T^{r+1}}{r+1} \cdot t \right]_T^{2.T} + \left[p \cdot \left(\frac{(2.T - t)^{r+2}}{(r + 2).(r + 1)} + \frac{T^{r+1}}{r + 1} \cdot t \right) \right]_T^{2.T} \quad (50)$$

$$L_k = \left(p\cdot\frac{T^{r+2}}{(r+2).(r+1)}\right) + \left(p\cdot\frac{T^{r+1}}{r+1}\cdot T\right) + p.\left(\frac{-(T)^{r+2}}{(r+2).(r+1)}\right) + p\cdot\frac{T^{r+1}}{r+1}\cdot T \tag{51}$$

$$L_k = p\cdot\frac{T^{r+1}}{r+1}\cdot T + p\cdot\frac{T^{r+1}}{r+1}\cdot T = \frac{2\cdot p\cdot T^{r+1}}{r+1} \tag{52}$$

So it is proven that the final result of the two integral methods above is the same.

The sixth case of nonlinear acceleration $f^+(t) = p.e^t$, with its mirror function $f^-(t) = f^+(T-t) = p.e^{T-t}$, the value of p is constant and the distance symbol is $L = L_g$. The symmetry function of nonlinear acceleration is $f_s(t) = p.e^t|_0^T + p.e^{2.T-t}|_T^{2.T}$, figure 6.

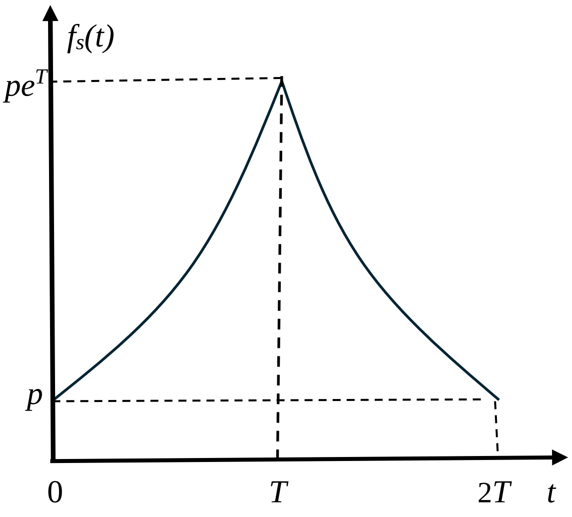

Figure 6. graph of nonlinear acceleration with its mirror function $f_s(t) = p.e^t|_0^T + p.e^{2.T-t}|_T^{2.T}$.

The result of calculating the total distance from time 0 to $2.T$ using the Goen distance formula is:

$$L_g = \int_0^{2.T}\int_0^t f_s(t).dt^2 = T\cdot\int_0^{2.T}\left(p.e^t|_0^T + p.e^{2.T-t}|_T^{2.T}\right).dt \tag{53}$$

$$L_g = \int_0^{2.T}\int_0^t f_s(t).dt^2 = T\cdot\int_0^{2.T}\left(p.e^t|_0^T + p.e^{2.T-t}|_T^{2.T}\right).dt \tag{54}$$

$$L_g = p.T.\left([e^t]_0^T - [e^{2T-t}]_T^{2.T}\right) = p.T.\left((e^T - e^0) - (e^0 - e^T)\right) \tag{55}$$

$$L_g = 2.p.T.(e^T - e^0) = 2.p.T.(e^T - 1) \tag{56}$$

Meanwhile, the results of the total distance calculation from time 0 to $2.T$ using the conventional formula L_k is:

$$L_k = \int_0^{2T}\int_0^t f_s(t).dt^2 = \int_0^{2T}\int_0^t \left(p.e^t|_0^T + p.e^{2.T-t}|_T^{2.T}\right).dt^2 \tag{57}$$

$$L_k = \int_0^{2T}\int_0^t p.e^t|_0^T.dt^2 + \int_T^{2T}\int_T^t \left(p.e^{2.T-t}\right).dt^2 \tag{58}$$

$$L_k = \int_0^T \left(p.(e^t - 1)\right).dt + \int_T^{2T}\left(p.(e^T - 1)\right).dt + \int_T^{2T} p\cdot\left(-e^{2.T-t} + e^T\right).dt \tag{59}$$

$$L_k = [p.(e^t - t)]_0^T + [p\cdot(e^T - 1)\cdot t]_T^{2T} + [p\cdot(e^{2.T-t} + e^T\cdot t)]_T^{2.T} \tag{60}$$

$$L_k = \left(p\cdot(e^T - T) - p\cdot e^0\right) + \left(p\cdot(e^T - 1)\cdot T\right) + p.(e^0 - e^T) + p\cdot e^T\cdot T \tag{61}$$

$$L_k = 2\cdot p\cdot e^T\cdot T - 2\cdot p\cdot T = 2.p.T.(e^T - 1) \tag{62}$$

So it is proven that the final result of the two integral methods above is the same.

The seventh case of nonlinear acceleration $f^+(t) = p.e^{r.t}$, with its mirror function $f^-(t) = f^+(T-t) = p.e^{r.(T-t)}$, the values of p and r are constant and the distance symbol is $L = L_g$. The symmetry function of nonlinear acceleration is $f_s(t) = p.e^{r.t}|_0^T + p.e^{r.(2.T-t)}|_T^{2.T}$, figure 7.

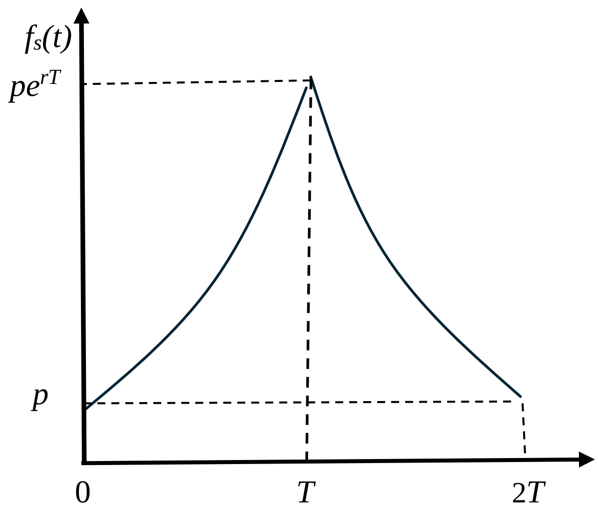

Figure 7. graph of nonlinear acceleration with its mirror function $f_s(t) = p.e^{r.t}|_0^T + p.e^{r.(2.T-t)}|_T^{2.T}$.

The result of calculating the total distance from time 0 to 2.*T* using the Goen distance formula is:

$$L_g = \int_0^{2\cdot T}\int_0^t f_s(t).dt^2 = T\cdot\int_0^{2\cdot T}\left(p.e^{r.t}\big|_0^T + p.e^{r.(2.T-t)}\big|_T^{2.T}\right).dt \tag{63}$$

$$L_g = T\cdot\left(\int_0^T (p.e^{r.t})dt + \int_T^{2\cdot T}\left(p.e^{r.(2.T-t)}\right)dt\right) \tag{64}$$

$$L_g = \left(\frac{p}{r}\right).T.\left([e^{r.t}]_0^T - [e^{2T-t}]_T^{2.T}\right) = \left(\frac{p}{r}\right).T.\left((e^{r.T}-e^0)-(e^0-e^{r.T})\right) \tag{65}$$

$$L_g = 2.\left(\frac{p}{r}\right).T.(e^{r.T}-e^0) = 2.\left(\frac{p}{r}\right).T.(e^{r.T}-1) \tag{66}$$

Compared with the general method, the result of calculating the total distance from time 0 to 2.*T* using the conventional formula L_k is:

$$L_k = \int_0^{2T}\int_0^t f_s(t).dt^2 = \int_0^{2T}\int_0^t\left(p.e^{r.t}\big|_0^T + p.e^{r.(2.T-t)}\big|_T^{2.T}\right).dt^2 \tag{67}$$

$$L_k = \int_0^{2T}\int_0^t p.e^{r.t}\big|_0^T.dt^2 + \int_T^{2T}\int_T^t\left(p.e^{r.(2.T-t)}\right).dt^2 \tag{68}$$

$$L_k = \int_0^T\left(p/r.(e^{r.t}-1)\right).dt + \int_T^{2T}\left(p/r.(e^{r.T}-1)\right).dt + \int_T^{2T} p/r\cdot\left(-e^{r.(2.T-t)}+e^{r.T}\right).dt \tag{69}$$

$$L_k = \left[p/r^2\cdot e^{r.t} - \frac{p}{r}\cdot t)\right]_0^T + \left[\left(\frac{p}{r}\cdot e^{r.T} - \frac{p}{r}\right)\cdot t\right]_T^{2T} + \left[p/r^2\cdot e^{2.T-t} + \frac{p}{r}\cdot e^{r.T}\cdot t\right]_T^{2.T} \tag{70}$$

$$L_k = \left(p/r^2\cdot e^{r.T} - \frac{p}{r}\cdot T - p/r^2\right) + \left(\frac{p}{r}\cdot e^{r.T} - \frac{p}{r}\right)\cdot T + p/r^2.(e^0 - e^T) + \frac{p}{r}\cdot e^{r.T}\cdot T \tag{71}$$

$$L_k = 2\cdot\frac{p}{r}\cdot e^{r.T}\cdot T - 2\cdot\frac{p}{r}\cdot T = 2.\left(\frac{p}{r}\right).T.(e^{r.T}-1) \tag{72}$$

So it is proven that the final result of the two integral methods above is the same. Of course, to overcome the limitations of the Goen distance formula, especially when dealing with non-symmetric acceleration functions, the solution is to continue to carry out the integration process twice on the non-symmetric acceleration function concerning time.

CONCLUSION

The Goen Distance Formula is a formula for calculating the distance where the acceleration function and its reflection are known. The distance formula states that calculating the distance where the acceleration function and its reflection will have a symmetrical pattern that can be solved simply by integrating once and then multiplying it by half the total time. The novelty of Goen's distance formula is that in finding the distance traveled, the acceleration function and its reflection in the form of a symmetrical acceleration function can be effective with one integral so that it is easier than the conventional method which must be integrated twice in calculating the distance of the object's motion. Therefore, a suggested area for future work on Goen's distance formula is that this distance formula can be integrated into software simulation with a shorter computational process in the case of an acceleration function that is symmetric concerning time.

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