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Lost Sales Inventory Model in Finite Planning Horizon

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Abstract.

This study extends a previous Economic Order Quantity (EOQ) model to include lost sales inventory and a finite planning horizon model. An exact algorithm is developed for finite planning horizon lost sales inventory condition. A practical approach is proposed to derive the optimal solution. The algorithm with lost sales inventory and finite planning horizon method improves the total cost of the inventory policy. Our analysis shows that the minimum total cost of the finite planning horizon method is always greater or equal to the infinite planning horizon method.

Keyword: *Inventory, Lost sales, Finite planning horizon*

1. INTRODUCTION

In classic inventory models it is common to assume that excess demand is backordered. However, studies analyzing customer behavior in practice show that most unfulfilled demand is lost or an alternative item / location is looked for in many retail or consumer environments. Lost sales case usually occurs in a perfectly competitive market. Lost sales happen when there is a shortage, demand is lost forever. Shortage causes lost sales cost and lost of good will for the distributor or retailer. According to Bijvank and Vis (2011), the worldwide out-of-stock rate is rather high with 7–8%. Their studied reveals that only 15% of the customers who observe a stock out will wait for the item to be on the shelves again, whereas the remaining 85% will either buy a different product (45%), visit another store (31%) or do not buy any product at all (9%).

Hadley and Whitin (1963) is one of the researchers that develop lost sales EOQ model. An overview of the lost sales inventory research is presented by Abad (2000) that studied optimal lot size for a perishable good under conditions of finite production and partial backordering and lost sale. Annadurai and Uthayakumar (2010) studied how to reducing lost-sales rate in inventory model with controllable lead time, and Bijvank and Vis (2011) studied a review about lost sales inventory.

The EOQ method has a weakness that may not be applicable in real cases, such as a number of order and order size that we get not in integer number and it will make difficult when the time was finite planning horizon that means the time planning period not continuously. Diponegoro and Sarker (2006) argued that an infinite horizon rarely occurred due to rapid technological development. They argued that the phenomenon can be frequently observed in high - technology product markets. Several researchers have developed alternative methods to solve the EOQ problem for finite planning horizon model. Kovalevand Ng (2008) developed an algorithm to derive an optimal number of orders for the classical EOQ with a finite discrete-time horizon. Li (2009) developed a new solution method to solve the problem discussed by Kovalev and Ng (2008). Wee et al. (2013) presented an alternative method to derive an EPQ with backorders using *Cauchy-Bunyakovsky-Schwarz Inequality*. This study presents a finite planning horizon Economic Order Quantity (EOQ) problem with quantity discount. We propose a method to solve EOQ problems with all unit quantity discounts for finite planning horizon model.

2. METHODOLOGY FOR MODEL DEVELOPMENT

2.1 Parameter in the Model

In this section, we formulate the finite horizon lot sizing model with lost sales inventory condition. The notations presented in Table 1 are used throughout the study.

Table 1 Notations used in this study

Parameters		**Units**
D	Demand rate per unit time	Units / year
σ	Standard deviation for demand rate per unit time	Units / cycle
L	Lead time	Year
α	Probability of stockout during the lead time	-
T	The length of a finite planning horizon	Year
z_{α}	Standard normal deviation	-
$f(z_{\alpha})$	Ordinates	-
$E(z_{\alpha})$	Partial expectation	-
x	Random variable demand during lead time	Units / cycle
Cost Parameters		
A	Ordering Cost	\$ / order
h	Holding Cost	\$ / unit / year
c_u	Shortages cost	\$ / unit / order
p	Unit price	\$ / unit
Objective Variables		
Q	Economic order quantity	Units / cycle
k	Amount of order	Times
r	Reorder point	Units / cycle
ss	Safety stock	Units / cycle

2.2 Model Assumption

The assumptions of this research are single-item inventory is considered, shortages are allowed and it will become lost sales, demand occurs have a normal distributions and continuous, orders can be placed at the beginning of any period, replenishment takes place instantaneously, ordering cost is known and constant, holding cost is known and constant, lead time is known and constant, purchase cost per order is given and no quantity discount, inventory storage capacity is assumed to be unlimited therefore having no capacity restrictions, and planning period time that used is in finite planning horizon.

2.3 Initial Model

The total cost of this inventory model consists of summation of purchasing cost, ordering cost, holding cost, and stock out cost Hadley and Whitin (1963). Because of purchasing cost is fixed and depend on demand, the total relevant cost only consist of ordering cost, holding cost, and stock out cost. This total relevant cost can be calculated as:

$$\boxed{TRC(Q,r) = Order\ cost + Holding\ cost + Stockout\ cost}$$

$$\boxed{TRC(Q,r) = A.\frac{D}{Q} + h\left(\frac{1}{2}Q + r - D_L + \int_r^{\infty}(x-r)f(x)dx\right) + \left(c_u\frac{D}{Q}\right)\int_r^{\infty}(x-r)f(x)dx}$$

$$= A.\frac{D}{Q} + h\left(\frac{1}{2}Q + r - D_L + \sigma_L[f(z_\propto) - z_\propto E(z_\propto)]\right) + \left(c_u\frac{D}{Q}\right)\sigma_L[f(z_\propto) - z_\propto E(z_\propto)] \quad (1)$$

The objective in the proposed model as in Hadley and Whitin (1963) is to minimize the expected value of the approximate total relevant cost. The cost equations are approximations because inventory levels and demands are treated as continuous instead of discrete quantities.

2.4 Proposed Model

This study modified model from Hadley and Whitin (1963) when planning period that used is in finite planning horizon. In this conditions, it is required that number of order must be in integer number, so that we modify the equation that proposed by Hadley and Whitin (1963) with multiplied by T because the planning period that used was in finite and change the objective variables by number of order (k) and reorder point (r).

To solve the proposed model and to find the optimal solution, the technique for solving the model development in this study will use proposed Hadley-Within algorithm solution approach to determine optimal number of order and reorder point level. Here are the *procedures approaches* that we used to solve and analyzed proposed model to determine optimum objective variables :

a. Calculate first a number of order using Wilson formula approach

$$k = T\sqrt{\frac{hD}{2A}}$$

b. Calculate probability inventory shortage using equation (6)

c. Calculate reorder point

$$r_i = DL + z_\propto \sigma\sqrt{L}$$

d. Calculate number of order lost sales inventory model using equation (5)

e. Calculate probability inventory shortage ($\propto_{i+1}$) and reorder point (r_{i+1})

f. Compare the value of reorder point (r_i) and (r_{i+1})

   If the value of reorder point (r_i) and (r_{i+1}) is not convergent, iterations back to step (d) by replacing (r_i) with (r_{i+1}) and continuous until r convergent, so that k and r can be determined. In this study, the value of reorder point is convergent if the difference of reorder point (r_i) and (r_{i+1}) are less than 1%.

g. After get k optimum from method above, determine Q. If amount of order not in integer number, use 2 two possibilities of $k = \lfloor k \rfloor$ or $\lceil k \rceil$ since the function of TRC (k,r) is convex with considering the optimal reorder point.

h. Determine order size per cycle

   From Wee, Wang et al. (2013), They proposed that the inventory policy of a constant batch size with one fill rate model is more cost efficient than that of variable batch sizes with variable fill rates.

i. Calculate total relevant cost using equation (4).

3. RESULTS AND ANALYSIS

3.1 Proposed Model for lost sale inventory

From Equation (1), the optimal total relevant cost for lost sales EOQ problem in finite planning period can be expressed as:

$$TRC(k,r) = \left[A.\frac{D}{Q} + h\left(\frac{1}{2}Q + r - D_L + \int_r^\infty (x-r)f(x)dx\right) + \left(c_u\frac{D}{Q}\right)\int_r^\infty (x-r)f(x)dx\right]T$$

Since $k = \frac{D}{Q}T$, equation above can be expressed as:

$$TRC(k,r) = Ak + h\left(\frac{DT^2}{2k} + rT - DLT + T\int_r^\infty (x-r)f(x)dx\right) + c_u k\int_r^\infty (x-r)f(x)dx \quad (2)$$

The optimal value for the k and r can be obtained by getting the first partial derivatives of the total cost function with respect to k and with respect to r equal to zero:

$$\frac{\partial TC(k,r)}{\partial k} = A - \frac{hDT^2}{2k^2} + Cu\int_r^\infty (x-r)f(x)dx = 0 \quad (3)$$

$$\frac{\partial TC(k,r)}{\partial r} = hT - (Cu\,k + h)\int_r^\infty f(x)dx = 0 \quad (4)$$

And solving simultaneously the linear system of equation (3) and (4), the expression for the optimal values of k and α can be obtained as:

$$k = T\sqrt{\frac{Dh}{2(A + Cu(\int_r^\infty (x-r)f(x)dx))}} \quad (5)$$

$$\alpha = \int_r^{\infty} f(x)dx = \frac{hT}{Cu\,k + hT} \tag{6}$$

3.2 Convexity test

To test the convexity of the total cost function, the Hessian matrix test is utilized. By using Hessian matrix, we can obtain the following equation:

$$[k \quad r]\begin{bmatrix} \frac{\partial^2 TRC(k,r)}{\partial k^2} & \frac{\partial^2 TRC(k,r)}{\partial k \partial r} \\ \frac{\partial^2 TRC(k,r)}{\partial r \partial k} & \frac{\partial^2 TRC(k,r)}{\partial r^2} \end{bmatrix}\begin{bmatrix} k \\ r \end{bmatrix} > 0 \tag{7}$$

Taking the second partial derivatives of the cost function with respect to *k* and to *r*, the following results are obtained:

$$\frac{\partial^2 TRC(k,r)}{\partial k^2} = \frac{hDT^2}{k^3} > 0 \tag{8}$$

$$\frac{\partial^2 TRC(k,r)}{\partial r^2} = (Cu\,k + h) \propto\, > 0 \tag{9}$$

$$\frac{\partial^2 TRC(k,r)}{\partial k \partial r} = \frac{\partial^2 TRC(k,r)}{\partial r \partial k} = Cu \ \propto > 0 \tag{10}$$

It is verified that the cost function is convex in *k* and in *r* since both second partial derivatives obtains a value greater than zero. Following the Hessian matrix test and the test for convexity, the condition must hold:

$$\left(\frac{\partial^2 TRC(k,r)}{\partial k^2}\right)\left(\frac{\partial^2 TRC(k,r)}{\partial r^2}\right) - \left(\frac{\partial^2 TRC(k,r)}{\partial k \partial r}\right) > 0 \tag{11}$$

Substituting equation (8), (9), (10) to the condition (11), and simplifying will yield a result:

$$\left(\frac{\partial^2 TRC(k,r)}{\partial k^2}\right)\left(\frac{\partial^2 TRC(k,r)}{\partial r^2}\right) - \left(\frac{\partial^2 TRC(k,r)}{\partial k \partial r}\right) = \frac{hDT^2(Cu\,k+h)\propto}{k^3} - Cu^2 \propto^2 > 0 \tag{12}$$

Since $\frac{hDT^2(Cu\,k+h)\propto}{k^3} - Cu^2 \propto^2 > 0$,

we can conclude that equation (2) is always positives thus the total relevant cost function is convex and there exist an optimal minimum values for both *k* and *r*. Since the total relevant cost function is convex, if the optimum *k* that resulted not in integer number, we have two options $\lfloor k \rfloor$ or $\lfloor k \rfloor + 1$. The chosen *k* is the one that gives the minimum total cost.

4. NUMERICAL EXAMPLES AND DISCUSSION

Assuming that D = 100,000 units per year, σ = 10,000 units per year, L = 1 months, σ_L = 10,000*√(1/12) = 2,887 units per cycle, A = $ 2,500 per order, h = $ 5 per unit per year, and cu = $ 100 per unit, and planning horizon time (T) = 1 year. To solve this problem, using Hadley-Within method approach for solving lost sales EOQ problem in finite planning horizon that describe in Section 2.4 *(procedures approaches).* From that method resulted economic number of order (k) = 5.54 times, safety stocks (ss) = 6,836 units, reorder point (r) = 15,169 units, given total relevant cost (TRC) = $ 124,681. Since amount of order (k) = 5.54 times, it is mean that we can't order 5.54 times in one year. From this condition, we have 2 possibilities to order 5 times or 6 times in a year. For **Scenario 1 (*k* = 5),** if k = 5, then Q = 20,000 units, and the other parameters follow the optimal solutions. From the parameters above resulted total relevant cost (TRC) = $ 125,159. For **Scenario 2 (*k* = 6)**, if k = 6, then Q = 16,666 – 16,667 units (Q = 16,666 units for 2 cycle, and Q = 16,667 units for 4 cycle), and the other parameters follow the optimal solutions. From the variable above resulted total relevant cost (TRC) = $ 124,966. To illustrate the relationship between ordering cost, holding cost, stock out cost, and total relevant cost for this case, it can be seen in Figure 1.

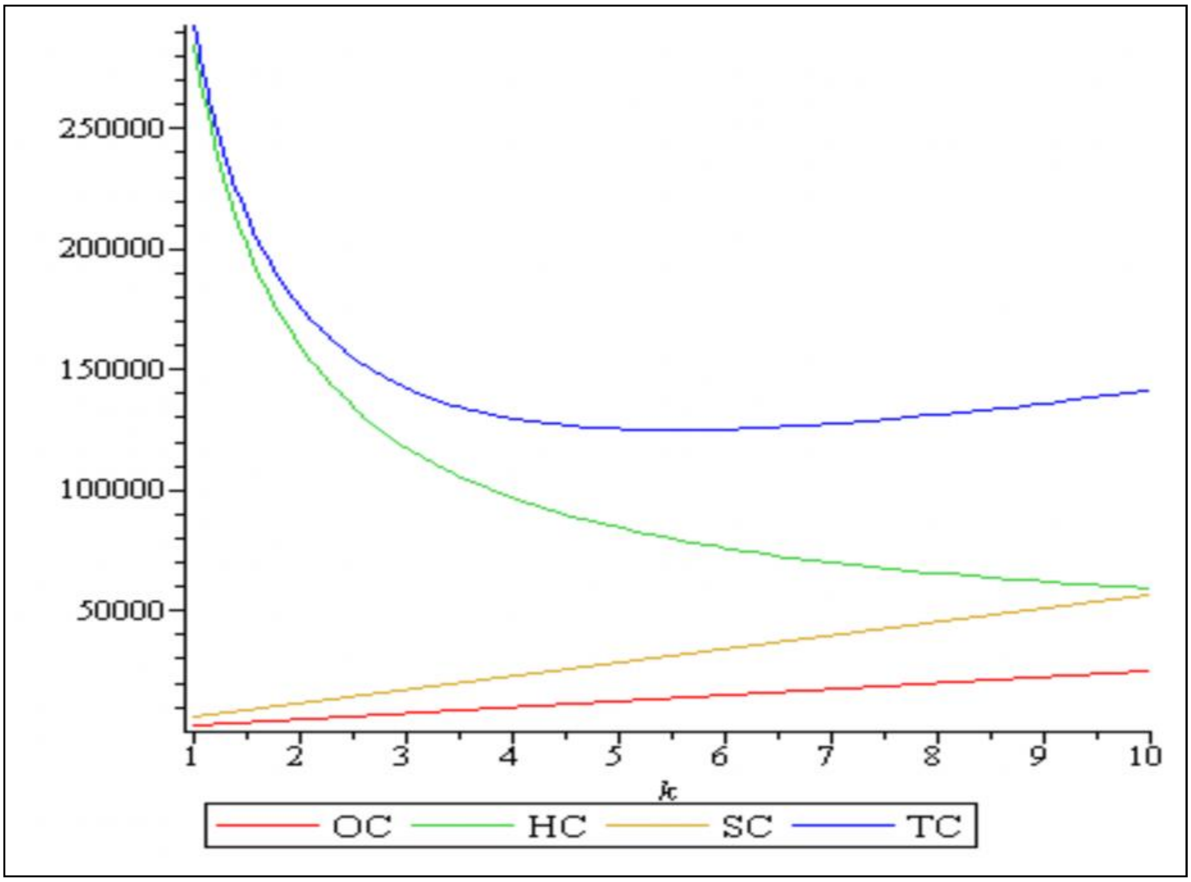

Figure 1.Relationship between ordering cost, holding cost, stockout cost, and total relevant cost for numerical example

Therefore, it's better to choose optimal number of order (k) = 6 in a year with the optimal order size (Q) = 16,666 – 16,667 units (Q = 16,666 units for 2 cycle, and Q = 16,667 units for 4 cycle), so that given minimum total relevant cost $ 124,938. From the result of total relevant cost, we can see that it has little difference (0.23%) compare with lost sales inventory that use infinite planning period, it means that the model that proposed in this model approaching the optimal value.

5. CONCLUSION

In this study, we propose a method to solve the lost sales inventory problem and finite planning horizon. From the analysis and numerical example, we can conclude that the minimum total cost of the finite planning horizon method is always greater or equal to the infinite planning horizon method. Future research can be done to consider multi items lost sales in finite planning horizon.

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