

Continuous Method: A Flexible Solution for Metal Length Expansion Measurement in the Modern Manufacturing Industry

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Abstract

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Measurement of metal length expansion is crucial in the modern manufacturing industry as it impacts product design, production, and performance. Current conventional method (Discrete Methods/DM) relies on an initial reference length (L0), which limits the flexibility of in-situ and continuous measurements. This study compares the accuracy and flexibility of DM with the Continuous Method (CM), an approach that potentially eliminates the need for L0 after the initial measurement. Using numerical simulations on five metals, this study analyzes calculations for estimating metal length expansion under various heating scenarios. The results show that both methods provide similar differences in metal length thermal expansion values (difference <0.0001 m), confirming their comparative accuracy. However, CM proves significantly more flexible as it allows the calculation of metal length thermal expansion estimates at arbitrary temperatures without the need to refer to L0, unlike DM, which requires L0 for consistency. This flexibility makes CM more practical for both incremental and real-time measurements, supporting the integration of automation into precision manufacturing processes. Implications for the manufacturing industry include increased efficiency of in-situ measurements, reduced reliance on initial data, and the potential for better integration with continuous quality control systems and Digital Twins.

1. INTRODUCTION

1.1 Background

The thermal expansion of metals, which is the dimensional change of a material due to temperature fluctuations, has a crucial impact on product design, production, assembly, and performance across various manufacturing sectors (Yakout *et al.*, 2020). If not accurately accounted for, thermal expansion can cause deformation, structural damage, functional failure, and even safety hazards. Industries such as steel construction and bridges require calculating the coefficient of thermal expansion to design expansion joints to prevent warping or cracking (Ma *et al.*, 2025). Similarly, the automotive industry must consider differences in expansion coefficients in machine components operating at extreme temperatures to avoid thermal stress and friction (Thomaz *et al.*, 2021). The railway industry relies heavily on rail expansion data to prevent dangerous buckling (Zhu *et al.*, 2016). Meanwhile, the energy sector designs pipeline expansion loops to accommodate the expansion of high-temperature fluids (Wankhede & Gawande, 2023). Even the exact aerospace industry must account for thermal expansion to maintain the structural integrity of aircraft at various altitudes and operating temperatures (Marble & Boles, 2022).

Currently, measurements of metal linear expansion typically use the Conventional Method (Discrete Method/DM), which assumes of a linear relationship between length change and temperature and requires knowledge of the initial reference length (L0). This dependence on L0 can be a constraint in in-situ and continuous measurement scenarios in modern manufacturing environments (Zheng *et al.*, 2015). Therefore, this study explores the Continuous Method (CM), an approach that offers greater flexibility because it does not always require L0 after the initial measurement.

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1.2 Research Objectives

The primary objectives of this study are to:

1. Analyze and compare the accuracy and flexibility of the Discrete Method (DM) and the Continuous Method (CM) in measuring metal linear expansion.
2. Evaluate the implications of the need for an initial reference length (L0) for both methods.
3. Assess the feasibility of implementing the Continuous Method in the context of automated, in-situ, and high-precision measurements in the modern manufacturing industry.

1.3 Research Benefits

This research is expected to provide several benefits, including:

1. Practical: Providing guidance for the manufacturing industry in selecting a more efficient and flexible metal linear expansion measurement method, especially in dynamic and automated production environments.
2. Scientific: Adding to the knowledge on material thermal characterization, particularly metal linear expansion modeling, by comprehensively comparing two different mathematical approaches.
3. Technology: Encouraging the development of more adaptive and precise thermal measurement systems that can be integrated with innovative manufacturing and Digital Twin technologies.

1.4. Brief Analysis Results

Preliminary analysis results indicate that both methods produce very similar linear expansion coefficients. However, the Continuous Method (CM) is significantly more flexible in calculating the final thermal expansion of the metal length because it does not always require an initial reference length; instead, it can use the metal length at any temperature as a reference. The consistency of the results between DM and CM is very high (difference < 0.0001 m), demonstrating the comparative accuracy of both methods. The flexibility of CM makes it more practical for stepwise and real-time measurements and is well-suited for automated integration in modern manufacturing processes (Subedi *et al.*, 2025).

2. LITERATURE REVIEW

Thermal expansion is a fundamental phenomenon in materials science that describes the dimensional change of a material in response to temperature changes (Kim *et al.*, 2025). In the context of metals, thermal expansion is highly relevant in engineering and industrial applications because it can directly impact on the structural integrity, functionality, and precision of products (Cheng *et al.*, 2021). The manufacturing industry, particularly those involved in precision materials and components, relies heavily on understanding and accurately measuring the coefficient of thermal expansion (CTE) of the materials used.

Traditionally, measuring the thermal expansion of metals relied heavily on the Discrete Method (DM). This approach is mathematically based on the assumption of a linear relationship between the change in length, the initial length, the coefficient of thermal expansion, and the temperature change (Tesfaye *et al.*, 2022). Although DM has long been used and proven to provide accurate results under controlled conditions, its dependence on the initial reference length (L0) can be a significant limitation in complex industrial environments. Modern manufacturing processes often involve gradual temperature changes and in-situ measurements, where obtaining or maintaining L0 may be impractical or

inefficient (Zheng *et al.*, 2015). Consequently, these methods present difficulties in continuous and automated measurements and can lead to error accumulation if the L0 reference is not strictly maintained (Peng *et al.*, 2022).

The need for more adaptive and flexible measurement methods has emerged in recent years. The Continuous Method (CM), derived from the differential approach, offers a promising alternative (Goenawan, 2025). The exponential formula generated by CM allows the calculation of linear expansion based on the metal length at a previous temperature point, rather than always from the initial reference length L0. This flexibility is particularly relevant for industrial applications requiring real-time and adaptive monitoring (Sprengel & Würschum, 2024). For example, in extrusion-based additive manufacturing, measuring the thermal expansion coefficient of fiber-filled polymer components has demonstrated the importance of continuous temperature-length data acquisition for understanding complex material behavior (Colón Quintana *et al.*, 2022).

Developing sensor technology and intelligent systems also supports the trend toward continuous measurement. Embedded sensors, such as fiber Bragg gratings, are increasingly being used to monitor dimensional and temperature changes in materials in situ, supporting the concepts of smart factories and Industry 5.0 (Ren *et al.*, 2025; Zhang *et al.*, 2023). Seamlessly, integrating linear expansion measurements into automated quality control systems is key to improving efficiency and reducing product defects. Furthermore, modern experimental accuracy has improved significantly, with techniques such as FIB-SEM-DIC that measure CTE at the micrometer scale (Robertson *et al.*, 2024). On the computational side, a quasiharmonic and density functional theory (QHA-DFT) approach for materials like Inconel 625 demonstrated how continuous modeling can accurately predict thermal behavior at high temperatures, validating the theoretical basis behind continuous methods (Shang *et al.*, 2024).

Thus, a comprehensive comparison between discrete and continuous methods, particularly in terms of their reference flexibility and adaptability to the requirements of modern manufacturing, is crucial. This research will explore how the Continuous Method can be a more efficient and practical solution for the need for precise and real-time metal linear expansion measurement, supporting innovation in manufacturing processes and material engineering (He *et al.*, 2025; Wang *et al.*, 2024).

3. RESEARCH METHOD

This research uses a comparative quantitative approach through numerical simulation. Five hypothetical metals (Metals A-E) with varying initial characteristics were used as samples. The research procedures include Calculation of the Coefficient of Linear Expansion: For each metal, DM and CM were used based on the initial length and temperature data. Length Expansion Simulation: Calculate the final metal length under various heating scenarios to compare the two methods, including testing the flexibility of the reference length. Data Collection: Simulation data are presented in tabular form (Tables 1-6). Validation Test: Comparing the accuracy and consistency between DM and CM based on the calculated results in measuring the estimated thermal expansion of the resulting metal length. Data Analysis: Data were analyzed quantitatively and comparatively to evaluate length changes, sensitivity to reference points, and consistency and dependence on the initial reference length of the metal.

This study analyzes and compares two approaches to measuring metal thermal expansion: the Discrete Method (DM) and the Continuous Method (CM). The primary focus

is accuracy, reference flexibility, and application feasibility in the modern manufacturing industry, particularly in automated, in-situ, and high-precision measurements (Zheng *et al.*, 2015; Qin *et al.*, 2023). This study uses a comparative quantitative approach, combining computational simulations, hypothetical data, and a literature review. The theoretical DM and CM models are used to calculate the coefficient of thermal expansion and the change in length of the metal under various temperature scenarios. Validation is carried out by comparing the calculated results of metal thermal expansion measurements using DM and CM methods (Tesfaye *et al.*, 2022; Colón Quintana *et al.*, 2022).

3.1. Research Design

This study adopts a comparative numerical simulation design. Five types of metals (Metals A-E) are used as simulation samples. Each metal has initial data in the form of an initial length (L_0), initial temperature (T_0), and length (L_1) at temperature T_1 . The objectives of this simulation are:

1. To calculate the coefficient of linear expansion (α_0) using two methods.
2. To analyze the change in length (L_q) under various temperature scenarios (from T_0 to T_1 , T_2 , and T_3).
3. To evaluate the flexibility of each method in using or removing dependence on the initial reference length (L_0).
4. To strengthen the research results, simulation data on metal characteristics ranging from metal A to E will be used as a comparative reference (Ren *et al.*, 2025; Shang *et al.*, 2024).

3.2. Research Procedure

3.2.1. Calculation of the Coefficient of Linear Expansion (α_0)

The initial stage involves determining the coefficient of linear expansion (α_0) for each metal type (A-E) using the L_0 , T_0 , T_1 , and L_1 data presented in Table 1. This was carried out using two methods:

a. Discrete Method (DM)

In DM, the thermal expansion estimation of a metal length is based on the assumption that the change in linear expansion (ΔL) is linearly proportional to the initial metal length (L_0) and the temperature change (ΔT). The formula is:

$$\Delta L = L_0 \alpha_0 \Delta T \quad \dots\dots\dots (1)$$

where α_0 is the coefficient of linear expansion of the metal. If $\Delta L = L_1 - L_0$ and $\Delta T = T_1 - T_0$, with the reference length (L_0) always have to be known beforehand, then the formula for the coefficient of linear expansion used in Table 1 is:

$$\alpha_0 = \frac{L_1 - L_0}{L_0(T_1 - T_0)} \quad \dots\dots\dots (2)$$

Similarly, from eq.(1), the estimated formula for thermal expansion of the metal length can be obtained, namely:

$$L_q = L_p + L_0 \alpha_0 (T_q - T_p) \quad \dots\dots\dots (3)$$

The formula in eq.(3) inherently requires knowledge of the reference length L_0 at the initial temperature (T_0) for each calculation. Even if measurements are made from the midpoint (e.g., calculating L_2 from L_1), the value of L_0 remains a crucial variable, as seen in eq.(3) where $p=1$ and $q=2$, then $L_2 = L_1 + L_0 \alpha_0 (T_2 - T_1)$.

b. Continuous Method (CM)

In contrast, CM is developed through a differential approach, where the change in linear expansion is linearly proportional to the metal length (L) and the change in temperature approaching zero (dT), $dL = L\alpha_0 dT$. Upon integration, this approach yields the exponential formula:

$$L_q = L_p e^{\alpha_0(T_q - T_p)} \dots\dots\dots (4)$$

A significant advantage of this formula is its ability to perform measurements without constantly referring to reference length L_0 . To calculate L_q from L_p , all that is needed is L_p itself, with no direct dependence on L_0 . This can be seen in the case of $q=2$ and $p=1$, where the linear expansion formula $L_2 = L_1 e^{\alpha_0(T_2 - T_1)}$ no longer requires the value of L_0 . Integration of $dL/L = \alpha_0 dT$, with lower limits L_0 , T_0 , and upper limits L_1 , T_1 , yields a continuous formula for the coefficient of linear expansion (Goenawan, 2025).

$$\alpha_0 = \frac{1}{(T_1 - T_0)} \ln\left(\frac{L_1}{L_0}\right) \dots\dots\dots (5)$$

The formula results from eq.(2) and (5) above will be used to measure the coefficients of linear expansion for metals A to E in the simulations in Table 1.

3.2.2. Simulation of Linear Expansion (L_q)

After the linear expansion coefficient α_0 for each metal was determined, simulations were performed to calculate the final length of the metal (L_q) under three different heating scenarios:

a. Heating from Temperature T_0 to T_2 :

The two methods used to estimate the thermal expansion from temperature T_0 to T_2 for this metal length use length L_0 as the initial reference point, and the results are recorded in Table 2. The Discrete Method (DM) formula is $L_2 = L_0 + L_0\alpha_0(T_2 - T_0)$, and the Continuous Method (CM) formula is $L_2 = L_0 e^{\alpha_0(T_2 - T_0)}$.

b. Heating from Temperature T_1 to T_2 :

Discrete Method (DM): The calculation is performed in two ways:

Using the reference L_0 : $L_2 = L_1 + L_0\alpha_0(T_2 - T_1)$, used for consistency comparison with L_0 .

Using reference L_1 : $L_2 = L_1 + L_1\alpha_0(T_2 - T_1)$, is used to evaluate flexibility without L_0 .

Continuous Method (CM): $L_2 = L_1 e^{\alpha_0(T_2 - T_1)}$, the results are recorded in Table 3, with a focus on comparing the DM calculations with reference points L_0 and L_1 , and the CM calculations with reference point L_1 .

c. Heating to Temperature T_3 from Various Reference Points (T_0 , T_1 , T_2):

1. The L_3 calculation from T_0 to T_3 in Table 4 follows a similar formula to the L_2 calculation from T_0 in Table 2.
2. The L_3 calculation from T_1 to T_3 in Table 5 is like the L_2 calculation from T_1 in Table 3, with DM calculated with and without reference point L_0 , and CM using reference point L_1 .

3. The L_3 calculation from T_2 to T_3 in Table 6 is like the previous calculation in number 2, with DM calculated with and without reference point L_0 , and CM using reference point L_2 . This was done to strengthen the findings regarding the comparative flexibility of the two methods over a broader and more gradual heating range.

For each calculation scenario with DM and CM, using a fixed reference point (L_0) or a dynamic reference point (L_1 , L_2), flexibility testing was used to determine which system was more sophisticated (Lenz *et al.*, 2022).

3.2.3. Data Collection and Validation Testing

The data used in this study are simulation calculations, not direct experimental data. The data are presented in tabular form, as attached in the "Analysis Results and Discussion" section (Tables 1 to 6). Each table includes the metal type, length, initial/final temperature, and the final length calculation results using Discrete and Continuous Methods. To test the accuracy and feasibility of the method in a real-world context, a comparative validation test was used between DM and CM in calculations to estimate the thermal expansion of metal lengths with five different characteristics (Qin *et al.*, 2023; Peng *et al.*, 2022; He *et al.*, 2025).

4. RESULTS AND DISCUSSION

This study analyzes two fundamental approaches to measuring metal linear expansion: the Conventional Method (Discrete Method/DM) and the Continuous Method (CM). The primary focus is to evaluate the flexibility and accuracy of each method, particularly the implications of the need for an initial reference length (L_0) in the context of a dynamic manufacturing industry (Schukraft *et al.*, 2022). Simulation calculations were performed to compare the two methods, from determining the linear expansion coefficient (α_0) to calculating the final metal length (L_q) over various temperature ranges.

4.1 Comparison Of Formulas and Coefficients of Linear Expansion

In the Discrete Method (DM), the linear expansion formula is based on the assumption of a linear relationship between the change in length (ΔL) and the initial length (L_0), the coefficient of linear expansion (α_0), and the temperature change (ΔT). In contrast, the Continuous Method (CM) is derived from the approximation of temperature changes $\left(\lim_{\Delta T \rightarrow 0} \Delta T\right)$ and length $\left(\lim_{\Delta L \rightarrow 0} \Delta L\right)$ that approach zero.

Simulations of determining the coefficient of linear expansion (α_0) with homogeneous metals A to E, presented in Table 1, show that both methods produce very similar α_0 values (Takamoto *et al.*, 2014). However, the Continuous Method tends to produce slightly smaller but consistent values. This slight difference likely stems from the different mathematical basis of the derivation, where CM accounts for infinitesimal length changes.

Table 1.

Determining the coefficient of linear expansion of metals using discrete and continuous methods

No.	Type	L_0 (m)	T_0 (C)	T_1 (C)	L_1 (m)	α_0 (C ⁻¹) $\times 10^{-5}$ DM	α_0 (C ⁻¹) $\times 10^{-5}$ CM
1	metal A	0,800	10	100	0,802	2,777778	2,774311
2	metal B	0,900	15	110	0,903	3,508772	3,502937
3	metal C	1,000	20	120	1,004	4,000000	3,992021
4	metal D	1,100	25	130	1,105	4,329004	4,319195
5	metal E	1,200	30	140	1,206	4,545455	4,534129

4.2. Analysis Of Final Metal Length (L_q) Measurement

Further testing was conducted to compare the ability of the two methods to calculate the final metal length (L_q) under various heating scenarios.

4.2.1. Heating the Metal Evenly from Temperature T_0 to T_2 (Table 2):

Simulation of calculating the metal length L_2 from temperature T_0 to T_2 using the length L_0 as the initial reference point shows that both the Discrete and Continuous Methods produce nearly identical values for the length L_2 resulting from the metal's thermal expansion. A minimal difference is observed in the last two decimal digits, indicating the consistency of the results of the two methods when starting from the reference length L_0 .

Table 2.

Finding the metal length (L_2) resulting from heating from T_0 to T_2 using the two methods

No.	Type	L_0 (m)	T_0 (C)	T_2 (C)	L_2 (m) DM	L_2 (m) CM
1	metal A	0,800	10	200	0,804222	0,804228
2	metal B	0,900	15	210	0,906158	0,906169
3	metal C	1,000	20	220	1,008000	1,008016
4	metal D	1,100	25	230	1,109762	1,109783
5	metal E	1,200	30	240	1,211455	1,211481

Based on the simulation of the length expansion calculation for metals A–E, both the Discrete Method (DM) and the Continuous Method (CM) produce the same final length values—very close L_2 , with a difference of 5 decimal places or less. For example, for metal B, using DM, the result is $L_2 = 0.906158$ m, and CM, which is $L_2 = 0.906169$ m.

4.2.2. Heating the Metal Evenly from Temperature T_1 to T_2 (Table 3):

When calculating the metal's linear expansion L_2 from temperature T_1 (using L_1 as the initial length), significant differences in the flexibility of the two methods begin to emerge (Schödel, 2008).

Table 3.

Finding the metal length (L_2) resulting from heating from T_1 to T_2 using two methods

No.	Type	L_1 (m)	T_1 (C)	T_2 (C)	L_2 (m) DM reference L_0	L_2 (m) DM reference L_1	L_2 (m) CM reference L_1
1	metal A	0,802	100	200	0,804222	0,804228	0,804228
2	metal B	0,903	110	210	0,906158	0,906168	0,906169
3	metal C	1,004	120	220	1,008000	1,008016	1,008016
4	metal D	1,105	130	230	1,109762	1,109784	1,109783
5	metal E	1,206	140	240	1,211455	1,211482	1,211481

In the Continuous Method (CM), the formula for the heated metal length, $L_2 = L_1 e^{\alpha_0(T_2 - T_1)}$, allows for the direct calculation of L_2 from the metal length L_1 and the temperature change between T_1 and T_2 , without requiring the initial reference length L_0 . Comparing Tables 2 and 3, calculating the metal length L_2 using the Continuous Method (CM) with the reference length L_1 is consistent with the results obtained from T_0 to T_2 .

In the Discrete Method (DM), to obtain consistent results from T_0 to T_2 , the calculation of the metal length L_2 starting from T_1 must still refer to L_0 , i.e., $L_2 = L_1 + L_0 \alpha_0 (T_2 - T_1)$. Suppose the Discrete Method attempts to calculate L_2 with only L_1 as the reference, that is, $L_2 = L_1 + L_1 \alpha_0 (T_2 - T_1)$. In that case, the results will be different and inaccurate compared

to the reference L_0 , as can be seen in the columns " $L_2(m)$ DM reference L_1 " versus " $L_2(m)$ DM reference L_0 " in Table 3 above.

4.2.3. Heating to Higher Temperatures (T_3) from Different Reference Points (Tables 4, 5, and 6): The same pattern was also confirmed in the L_3 calculation simulations.

Table 4 shows that L_3 calculations from temperature T_0 to T_3 yield values similar to those of both methods, like the findings in Table 2.

Table 4.

Finding the metal length (L_3) resulting from heating from T_0 to T_3 using two methods

No.	Type	$L_0(m)$	$T_0(C)$	$T_3(C)$	$L_3(m)$ DM	$L_3(m)$ CM
1	metal A	0,800	10	300	0,806444	0,806462
2	metal B	0,900	15	310	0,909316	0,909349
3	metal C	1,000	20	320	1,012000	1,012048
4	metal D	1,100	25	330	1,114524	1,114587
5	metal E	1,200	30	340	1,216909	1,216986

Tables 5 and 6 demonstrate the superior flexibility of the Continuous Method. When calculating the length of metal L_3 from L_1 , as shown in Table 5 below:

Table 5.

Finding the metal length (L_3) resulting from heating from T_1 to T_3 using two methods

No.	Type	$L_1(m)$	$T_1(C)$	$T_3(C)$	$L_3(m)$ DM reference L_0	$L_3(m)$ DM reference L_1	$L_3(m)$ CM reference L_1
1	metal A	0,802	100	300	0,806444	0,806456	0,806462
2	metal B	0,903	110	310	0,909316	0,909337	0,909349
3	metal C	1,004	120	320	1,012000	1,012032	1,012048
4	metal D	1,105	130	330	1,114524	1,114567	1,114587
5	metal E	1,206	140	340	1,216909	1,216964	1,216986

Alternatively, calculate the length of metal L_3 from L_2 as shown in Table 6 below:

Table 6.

Finding the metal length (L_3) resulting from heating from T_2 to T_3 using two methods

No.	Type	$L_2(m)$ DM	$L_2(m)$ CM	$T_2(C)$	$T_3(C)$	$L_3(m)$ DM reference L_0	$L_3(m)$ DM reference L_2	$L_3(m)$ CM reference L_2
1	metal A	0,804222	0,804228	200	300	0,806444	0,806456	0,806462
2	metal B	0,906158	0,906169	210	310	0,909316	0,909337	0,909349
3	metal C	1,008000	1,008016	220	320	1,012000	1,012032	1,012048
4	metal D	1,109762	1,109783	230	330	1,114524	1,114566	1,114587
5	metal E	1,211455	1,211481	240	340	1,216909	1,216961	1,216986

The results show that the Continuous Method (CM) can directly use L_1 or L_2 as the initial length, resulting in a metal length value L_3 consistent with the calculation from L_0 . In contrast, the Discrete Method will produce discrepancies if it does not explicitly use L_0 as the reference, confirming its dependence on the absolute initial length

4.3. Implications For the Manufacturing Industry

From this analysis, it can be concluded that the Continuous Method (CM) offers significantly greater flexibility in measuring metal linear expansion than the Discrete

Method (DM). In the context of the manufacturing industry, this flexibility is crucial (Monroe *et al.*, 2015):

- a. In-Situ and Repeatable Measurements: In manufacturing processes, metals often undergo multiple heating and cooling stages. With CM, length measurements can be taken at any point in the process, without knowing the initial metal length at standard conditions (L_0). This enables more efficient in-situ and repeatable measurements on the production line.
- b. Reduced Dependence on Initial Data: DM's reliance on L_0 can be a limitation if L_0 cannot be measured precisely in every production cycle or if L_0 information is missing. CM addresses this issue by enabling accurate measurements based on data available during measurement.
- c. Potential for Automation Integration: MK's flexibility makes it more suitable for automated measurement systems and continuous quality control. Sensors can measure length at arbitrary temperatures, and the system can directly calculate the expansion or final length without resetting to L_0 . This supports the implementation of concepts such as Digital Twins for thermal deformation monitoring (Fu *et al.*, 2024).

Although the coefficient of metal linear expansion (α_0) values produced by both methods are very close, the Continuous Method's ability to perform calculations from a "flexible" reference point makes it a more practical and efficient choice for dynamic industrial manufacturing applications. This allows for more adaptive and accurate monitoring of metal dimensional changes during production, supporting improved quality and efficiency.

4.4. Discussion

4.4.1 Accuracy and Flexibility

The Continuous Method (CM) offers calculations equivalent to the Discrete Method (DM), perhaps even more mathematically accurate because CM uses a continuous differential approach. This allows for flexible measurements of metal linear expansion, especially when temperature changes are gradual. It eliminates the need to refer to L_0 each time a calculation is performed, simply using the metal's length before expansion. This contrasts with DM, which always requires information about the initial reference length L_0 to avoid error accumulation.

4.4.2. Data Consistency

The calculation results for metal expansion measurements in Tables 2, 3, 4, 5, and 6 consistently show that the difference between DM and CM values is minimal (< 0.0001 m), indicating that CM is not only consistent but also allows for easier repeatability of measurements under changing conditions.

4.4.3. Implications for the Manufacturing Industry

In manufacturing industry practice, Thermal expansion processes often occur gradually, and temperature changes are not always immediate from the initial reference temperature (T_0). CM enables the integration of automated, continuous measurements of metal length expansion, such as temperature and length sensors that can directly calculate new length predictions at any time, without manual intervention and recalculation of the initial reference length (L_0).

4.4.4. Real-Time Measurement

Modern literature supports the trend toward continuous, real-time measurements of the Coefficient of Thermal Expansion (CTE). For example, using embedded sensors such as fiber Bragg gratings in Fused Deposition Modeling (FDM) 3D printing demonstrates the importance of capturing continuous temperature–length data (Colón *et al.*, 2022). Other studies highlight that continuous measurements, for example, in metal or composite materials, enable better quality control and rapid detection of distortions (Kul *et al.*, 2024).

4.4.5. Modern Experimental Accuracy

Modern methods such as FIB SEM DIC are capable of measuring CTE at micro-units with high accuracy (Robertson *et al.*, 2024), while QHA DFT for Inconel 625 demonstrates how a theoretical continuous approach can be suitable for high-temperature simulations (Shang *et al.*, 2024).

Therefore, this discussion can be summarized as follows:

- a. Continuous Method (CM) have proven more flexible for stepwise and real-time measurements because they do not require an initial reference length (L_0) after the first step and are suitable for automated integration in modern manufacturing processes.
- b. Discrete Method (DMD), while still accurate, are less practical in stepwise measurement schemes because they depend continuously on the initial reference length L_0 .
- c. CM can be more relevant and practical in modern industrial applications, particularly in precision manufacturing and structural monitoring.

5. CONCLUSION

This study successfully analyzed and compared the accuracy and flexibility of the Discrete Method (DM) and the Continuous Method (CM) in measuring metal linear expansion, focusing on the implications of the need for an initial reference length (L_0) for application in the modern manufacturing industry. Numerical simulation results show that both methods produce similar values for the coefficient of linear expansion (α_0) and estimates of the thermal expansion of the metal length (L_q), with a difference of less than 0.0001 meters, confirming their comparative accuracy.

However, the key finding of this study is the superior flexibility of the Continuous Method. Unlike DM, which inherently requires knowledge of L_0 for every consistent calculation, CM allows for calculating thermal expansion from a metal length at an arbitrary reference temperature without needing to refer back to L_0 after the initial measurement. CM's ability to directly calculate the metal length at a previous temperature point (L_p) makes it more practical for incremental and real-time measurements.

The practical implications of CM's flexibility for the manufacturing industry are significant. This method supports increased efficiency of in-situ and repeatable measurements in production lines, reduces dependence on initial data that is sometimes difficult to maintain precisely or easily lost, and facilitates the integration of automation in continuous quality control systems and Digital Twin concepts for thermal deformation monitoring. CM has proven to be more relevant and practical for precision manufacturing and structural monitoring in modern, dynamic industrial environments. Therefore, this study concludes that the Continuous Method offers a more flexible, adaptive, and efficient solution for precise and real-time metal linear expansion measurements in the present and future.

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